OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4736

Decision Mathematics 1

Tuesday **7 JUNE 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Question **4**.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 5 printed pages, 3 blank pages and an insert.

1 **(a)** **(i)** Use the first-fit decreasing method to pack these weights, in kg, into bags that can each hold a maximum of 10 kg. [3]

2 7 5 3 3 4 3 2 8 3

(ii) Find a packing that uses fewer bags. [1]

(b) The order of a particular algorithm is a cubic function of the number of input values. It takes 4 seconds for the algorithm to process 100 input values. Approximately how many seconds will it take the algorithm to process 500 input values? [2]

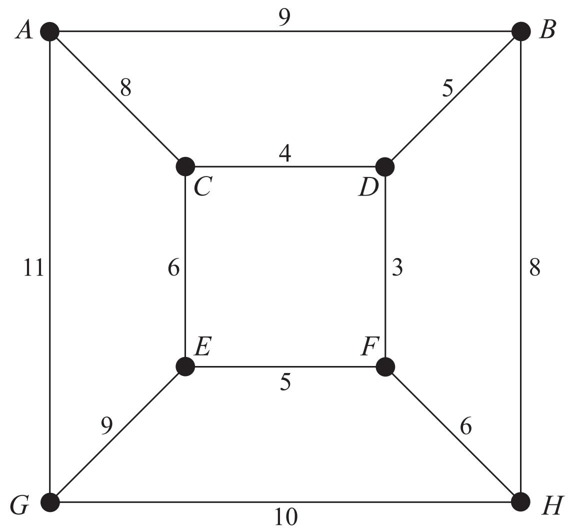
2 A *simple* graph is one which has no repeated arcs and no arc that joins a vertex to itself.

(i) Draw a simple graph that connects four vertices using five arcs. [1]

(ii) Explain why, in **any** graph, there must be an even number of odd vertices. [2]

(iii) By considering the orders of the vertices, show that there is only one possible simple graph that connects four vertices using five arcs. [3]

3 This diagram shows a network.



(i) Obtain a minimum connector for this network. Draw your minimum connector, state the order in which the arcs were chosen and give their total weight. [3]

(ii) Use the nearest neighbour method, starting from vertex A, to find a cycle that passes through every vertex. [2]

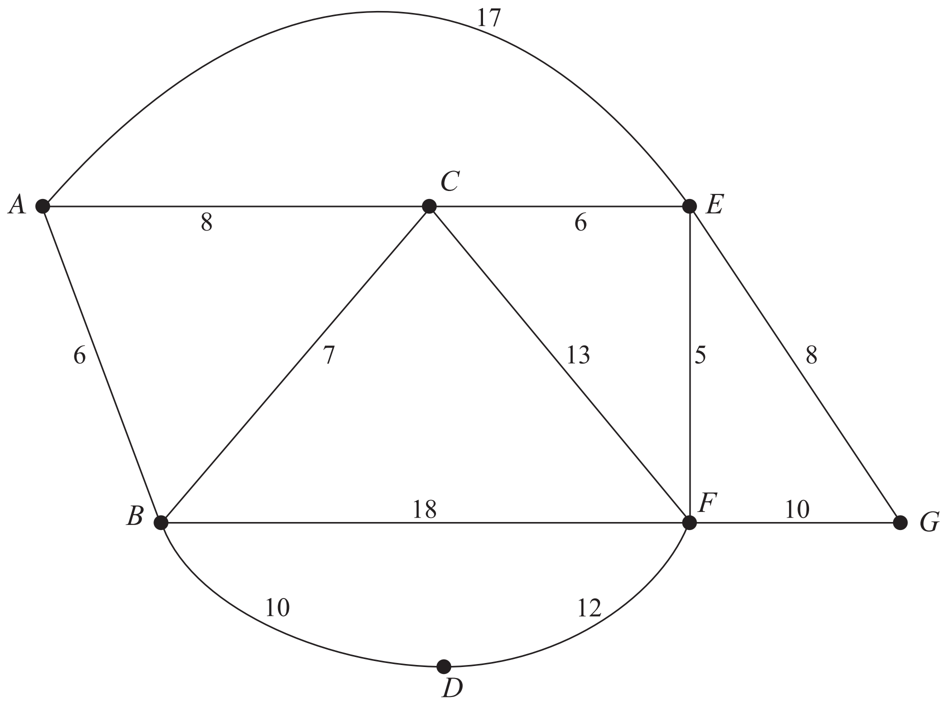
The network represents a cubical die, with vertices labelled A to H, and faces numbered from 1 to 6 in such a way that the numbers on each pair of opposite faces add up to 7. When two faces meet in an edge, the sum of the numbers on the two faces is recorded as the weight on that edge.

(iii) **(a)** List the vertices of each of the two faces that meet in the edge AG. [1]

(b) What number is on the face $ACEG$? [1]

(c) Which face is numbered 3? [1]

4 [Answer this question on the insert provided.]



In this network the vertices represent towns, the arcs represent roads and the weights on the arcs show the lengths of roads in kilometres.

(i) Use Dijkstra's algorithm on the diagram in the insert to find the length of the shortest path from A to each of the other vertices. You must show your working, including temporary labels, permanent labels and the order in which the permanent labels were assigned. Find the route of the shortest path from A to G. [5]

The total weight of the arcs is 120 kilometres.

(ii) By using an appropriate algorithm, find the length of a shortest route that uses **every** road starting and ending at A. You should explain your method. [3]

(iii) Find the length of a shortest route that uses **every** road starting at A and ending at G. You should explain your method. [2]

5 Consider the following algorithm which is to be applied to a list of numbers.

Step 1	Let $N = 0$, $T = 0$ and $S = 0$.
Step 2	Input the first number in the list and call it X. Delete the first number from the list to give a list that has one number fewer than before.
Step 3	Increase N by 1, increase T by X and increase S by X^2.
Step 4	If there are still numbers in the list then go back to Step 2. Otherwise go to Step 5.
Step 5	Calculate $M = (T$ divided by $N)$. Calculate $V = (S$ divided by $N) - (M$ squared$)$. Calculate $D = \sqrt{V}$.
Step 6	Output M and D.

(i) Apply the algorithm to this list.

3 6 5 7 3

Record in a table the values of X, N, T and S at each pass through Step 3 and give the output values. [5]

(ii) Write down the number of additions and the number of multiplications that are done in Step 3 for a list of five numbers. Hence find the total number of arithmetic operations (additions, multiplications, divisions, subtractions and square roots) that are done in Step 3 and Step 5 when applying the algorithm to a list of five numbers. [2]

(iii) Find an expression for the total number of arithmetic operations that are done in applying the algorithm to a list of n numbers. [2]

(iv) The total number of arithmetic operations can be used as a measure of the run-time for the algorithm. If it takes approximately 2 seconds to apply the algorithm to a list of 1000 numbers, approximately how long will it take to apply the algorithm to a list of 5000 numbers? [1]

6 **(i)** Represent the linear programming problem below by an initial Simplex tableau. [2]

$$\begin{array}{ll} \text{maximise} & P = 15x - 4y - 4z, \\ \text{subject to} & 10x - 4y + 8z \leqslant 40, \\ & 10x + 6y + 9z \leqslant 72, \\ & -6x + 4y + 3z \leqslant 48, \\ \text{and} & x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0. \end{array}$$

(ii) Perform **one** iteration of the Simplex algorithm and write down the values of x, y, z and P that result from this iteration. [6]

(iii) Perform one further iteration of the Simplex algorithm and find the values of x, y, z and P at the optimum point. [6]

7 Three types of A4 paper are sold in a shop.

Pack type	Description	Cost per pack
X	White 80 g	70p
Y	White 50 g	80p
Z	Coloured 80 g	50p

From past experience, it is known that each week the shopkeeper should order:

at least 200 packs of type X;
no more than twice as many packs of type Y as packs of type X;
at least as many packs of type Y as packs of type Z;
no more than 50 packs of type Z;
at least 220 packs of 80 g paper;
at least 300 packs of white paper.

The problem is to achieve this at the minimum cost.

Let x be the number of packs of type X ordered, y be the number of packs of type Y ordered and z be the number of packs of type Z ordered. The variables x, y and z must all be non-negative integers.

(i) Identify the objective. Explain why $y \leqslant 2x$ is a constraint and find the other constraints for the problem. [6]

The shopkeeper orders exactly 50 packs of type Z, because they are the cheapest.

(ii) **(a)** Reformulate the linear programming problem in terms of the variables x and y. Show the feasible region graphically. [5]

(b) Write down the x and y values at each vertex of the feasible region. Hence find the values of x and y for which the cost is minimised and state the minimum cost in this situation. [5]

(iii) Show that ordering 50 packs of type Z does **not** give the minimum cost solution for the original problem. [2]

BLANK PAGE

BLANK PAGE

BLANK PAGE

Candidate Name	Centre Number	Candidate Number

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4736

Decision Mathematics 1

INSERT for Question 4

Tuesday **7 JUNE 2005** Afternoon 1 hour 30 minutes

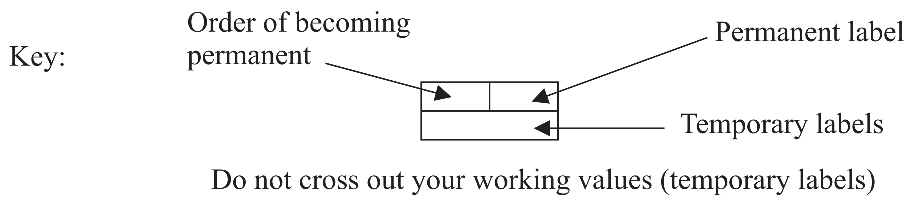
INSTRUCTIONS TO CANDIDATES

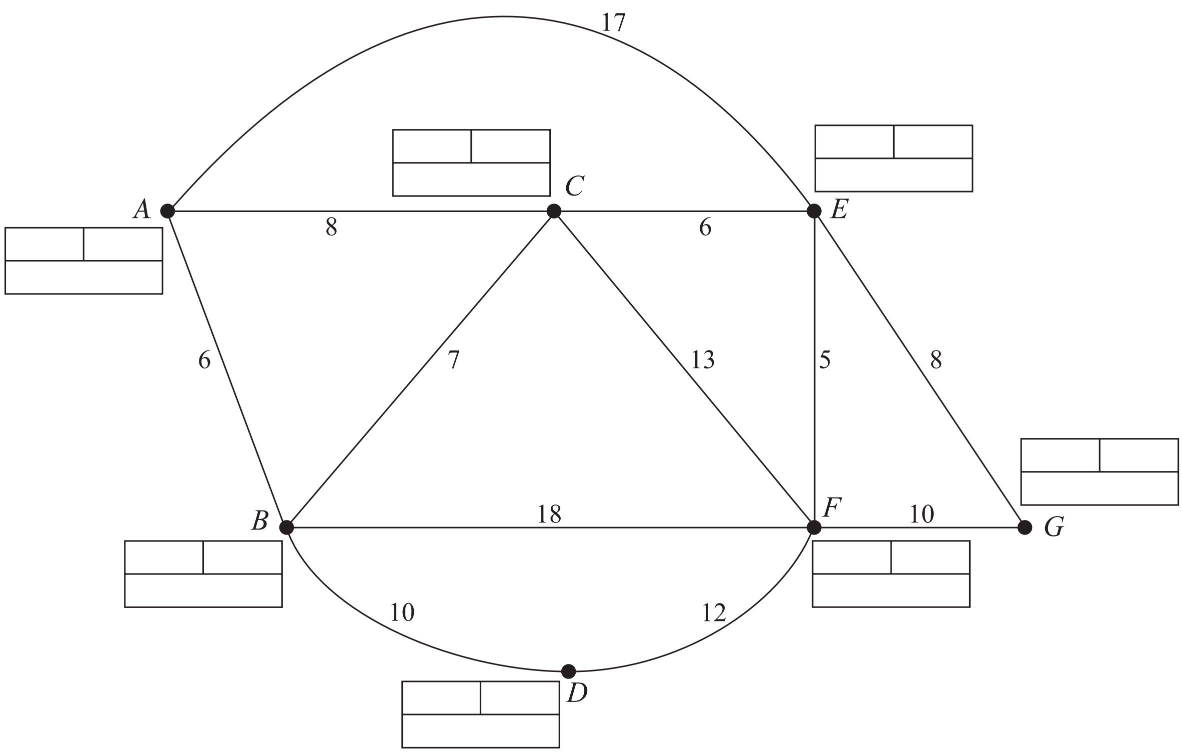
- This insert should be used to answer Question **4**.
- Write your name, centre number and candidate number in the spaces provided at the top of this page.
- Write your answers to Question **4** in the spaces provided in this insert, and attach it to your answer booklet.

This insert consists of 2 printed pages.

[Turn over

4 **(i)**





Vertex	B	C	D	E	F	G
Length of shortest path from A						

Route of shortest path from A to G ....

(ii) ....

....

....

Length of shortest route that uses **every** road starting and ending at A .... km

(iii) ....

....

....

Length of shortest route that uses **every** road starting at A and ending at G .... km

BLANK PAGE