OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2603(A)

Pure Mathematics 3

Section A

Monday **10 JANUARY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

NOTE

- This paper will be followed by **Section B: Comprehension**.

This question paper consists of 4 printed pages.

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1 **(a)** Express $\dfrac{5}{(x-2)(x^2+1)}$ in the form $\dfrac{A}{x-2}+\dfrac{Bx+C}{x^2+1}$, where A, B and C are constants to be determined. [5]

(b) Write down the exact values of $\sin 30°$ and $\cos 30°$. Express $\sin(x+30°)$ in terms of $\sin x$ and $\cos x$.

Hence solve the equation

$$\sqrt{3}\sin x+\cos x=1 \qquad \text{for } -180° < x < 180°.$$ [6]

(c) A curve has parametric equations

$$x = 1+t^2,\ y = t^3.$$

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. Hence find the gradient of the curve at the point $(2, 1)$. [4]

2 **(i)** Using a small angle approximation for $\cos x$, show that, for small values of x,

$$x\sec^2 x \approx x(1-\tfrac{1}{2}x^2)^{-2}.$$

Use a binomial expansion to show that, for small x,

$$x\sec^2 x \approx x+x^3.$$ [4]

(ii) Use this result to evaluate $\displaystyle\int_0^{0.1} x\sec^2 x\,\mathrm{d}x$ approximately. [2]

(iii) By differentiating $\dfrac{\sin x}{\cos x}$, show that the derivative of $\tan x$ is $\sec^2 x$. [3]

(iv) Use integration by parts, together with the result $\displaystyle\int \tan x\,\mathrm{d}x = -\ln\cos x + c$, to evaluate

$$\int_0^{0.1} x\sec^2 x\,\mathrm{d}x.$$

Comment on the accuracy of the approximate result you found in part **(ii)**. [6]

3 **(i)** Show that the solution to the differential equation

$$\frac{dy}{dx} = -\frac{4x}{y},$$

with $y = 2$ when $x = 0$, is

$$4x^2 + y^2 = 4.$$ [4]

(ii) Show that parametric equations for the curve $4x^2 + y^2 = 4$ are

$$x = \cos\theta,\ y = 2\sin\theta.$$

What sort of curve is this? [3]

Fig. 3 shows part of the curve, and a point P on it. The tangent at P meets the x-axis at Q. PQ makes an angle β with the x-axis, and angle POQ is α, where $0° < \alpha < 90°$.

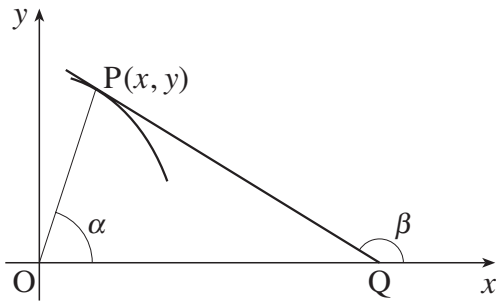

Fig. 3

(iii) Express $\tan\alpha$ in terms of x and y. Hence, using the differential equation given in part **(i)** and the fact that $\frac{dy}{dx} = \tan\beta$, show that

$$\tan\alpha \tan\beta = -4.$$ [2]

(iv) You are given that, for a particular point on the curve, $\beta = 2\alpha$. Show that, for this point,

$$\tan^2\alpha = 2.$$

Find the value of α, giving your answer to the nearest degree. [6]

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4 Fig. 4 shows the roof of a house. The coordinates of points A, B, C, D, E and F with respect to axes Ox, Oy and Oz are as shown in the diagram. ABCDE is a plane. All lengths are in metres.

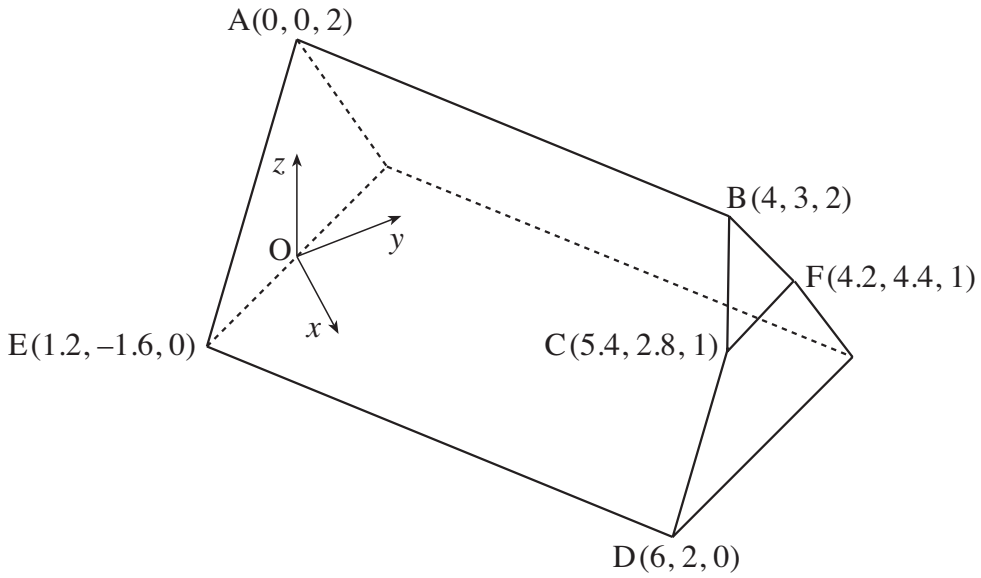

Fig. 4

(i) Write down the vectors $\overrightarrow{BC}$ and $\overrightarrow{BF}$. [1]

(ii) Show that triangle BCF is isosceles. [2]

(iii) Verify that the vector $\mathbf{n}_1 = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ is normal to the plane BCF. Deduce the equation of the plane BCF. [5]

(iv) Verify that the cartesian equation of the plane through A, B and D is

$$3x - 4y + 5z = 10.$$

Write down a vector $\mathbf{n}_2$ normal to this plane. [3]

(v) Find the angle between the planes BCF and ABCDE. [4]

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