OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2645

Discrete Mathematics 1

Wednesday **12 JANUARY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

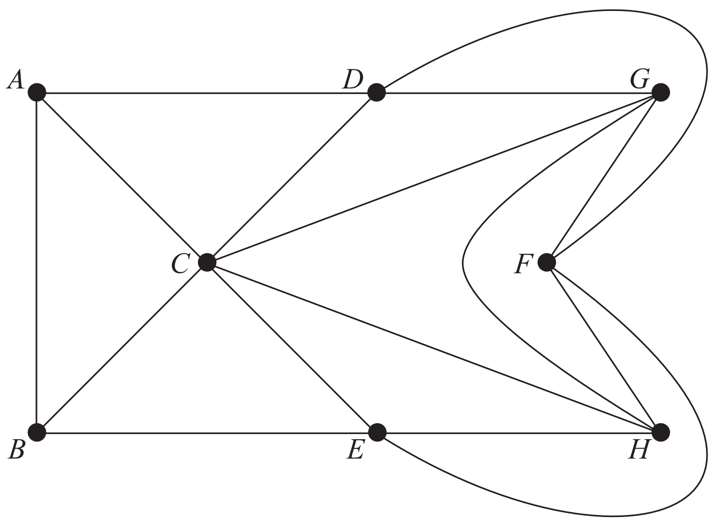
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 5 printed pages, 3 blank pages and an insert.

1 **(i)** How is the number of arcs in a graph related to the order of the vertices? [1]

The diagram shows a graph.

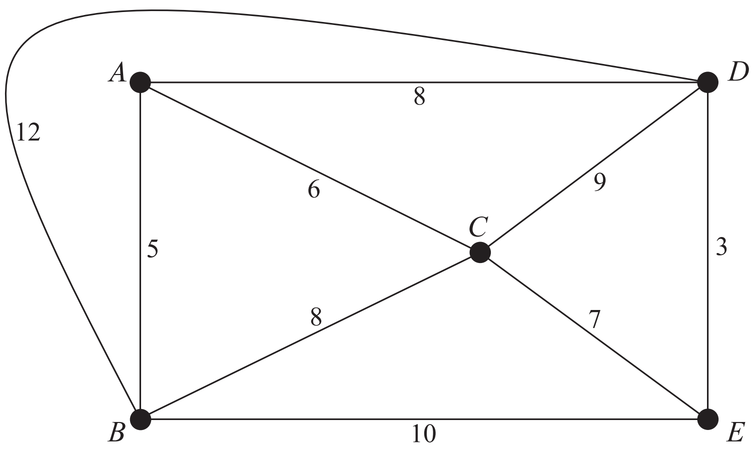


(ii) Which arc, if removed, would leave an Eulerian graph? [1]

A Hamiltonian cycle is a cycle that passes through every vertex.

(iii) How many arcs are used in a Hamiltonian cycle for this graph? [1]

2 The diagram shows a network. The weights on the arcs represent distances in miles. The direct path between any two adjacent vertices is always shorter than any indirect path.



(i) Use the nearest neighbour method to find a route that starts at A and visits every other vertex once before returning to A. Calculate the length of this route. [3]

(ii) By deleting vertex C and all arcs connected to C, find a lower bound for the length of the shortest cycle that visits every vertex of the original network. [2]

3 Jenny needs to cut the following lengths, in metres, from rolls of wallpaper which are each ten metres long.

6 5 5 7 8 4 5 2

(i) Use the first-fit decreasing algorithm to show that Jenny can cut these from five rolls of paper. [2]

Jenny has allowed 50 cm on each of the eight lengths for matching the pattern. If she uses plain paper the lengths that she needs to cut can be reduced to those listed below.

5.5 4.5 4.5 6.5 7.5 3.5 4.5 1.5

(ii) Show that if Jenny uses plain paper she can cut these from four rolls of paper. [2]

Jenny finds out that the plain paper is also available in longer rolls. The longer rolls are a whole number of metres long.

(iii) How long must these rolls be for her to need to cut only three rolls of paper? Show that she can cut the lengths she needs from three rolls of this length. [3]

4 [Answer this question on the insert provided.]

Six stations are connected by rail links. The lengths, in kilometres, of the links are shown in the table.

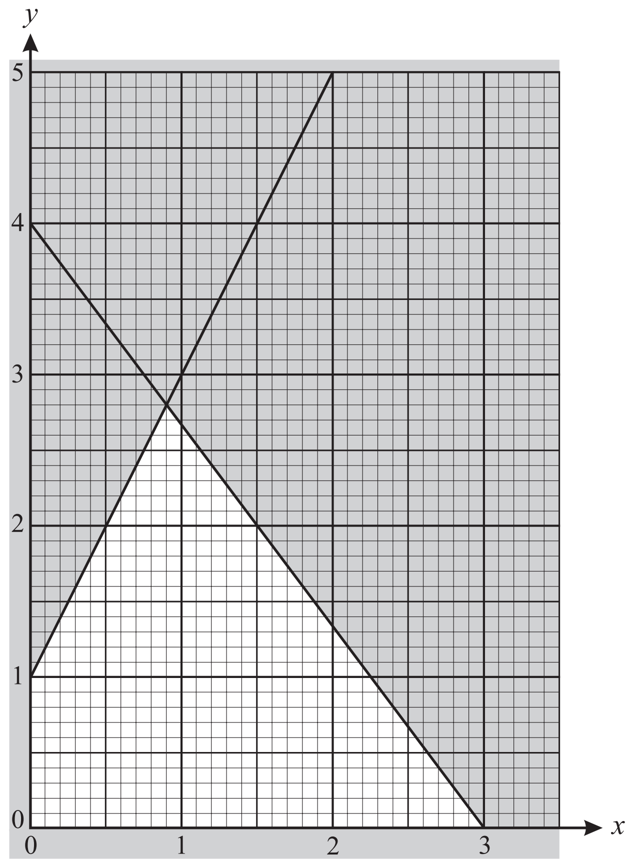
	A	B	C	D	E	F
A	–	–	5	–	–	–
B	–	–	6	8	15	8
C	5	6	–	9	23	20
D	–	8	9	–	–	–
E	–	15	23	–	–	4
F	–	8	20	–	4	–

(i) On the insert, draw a network to represent the information given in the table. [2]

(ii) Use Dijkstra's algorithm to find the shortest route from A to E. Show your working clearly, including the values of the permanent labels and the order in which the permanent labels were assigned to the vertices. [4]

(iii) By considering pairings of odd nodes, find the length of the shortest route that starts and ends at A and uses every link at least once. [3]

5 The constraints of a linear programming problem are represented by the graph below. The feasible region is the unshaded region, including its boundaries.



(i) Write down four inequalities that define the feasible region. [3]

The objective is to maximise $P = 5x + 3y$.

(ii) Obtain the coordinates of the vertices of the feasible region and hence find the values of x and y that maximise P, and the corresponding maximum value of P. [6]

The objective is changed to maximise $Q = ax + 3y$.

(iii) For what set of values of a is the maximum value of Q equal to 3? [3]

6 [Answer this question on the insert provided.]

Consider the following algorithm.

Step 1 Input a positive integer N.
If 2 divides into N exactly, go directly to step 4.
Otherwise, set $D = 3$.

Step 2 If D^2 is greater than N, go directly to step 6.

Step 3 If D divides into N exactly, go directly to step 5.
Otherwise, increase D by 2 and then go back to step 2.

Step 4 Output '2' and stop.

Step 5 Output the value of D and stop.

Step 6 Output the value of N and stop.

(i) Work through the algorithm to find the output for each of the integers from 30 to 37. Use the table in the insert. [6]

(ii) Why can the output never be 6 and why can it never be 9? [2]

(iii) What is the relationship of the output to the input? [2]

(iv) Explain how you know that the algorithm will always terminate. [2]

7 Consider the linear programming problem:

maximise $P = x - 4y - 2z$,

subject to $2x + 3y + z \leqslant 12$,
$x + 2y + 4z \leqslant 8$,

and $x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$.

(i) Using slack variables, s and t, express the non-trivial constraints as two equations. [1]

(ii) Represent the problem as an initial Simplex tableau. Perform **one** iteration of the Simplex algorithm. Write down the values of x, y, z and P resulting from this iteration. [8]

(iii) The value 8 in the second constraint is changed to a new value c, where $c < 6$. Perform **one** iteration of the Simplex algorithm. Write down expressions for x, y, z and P resulting from this iteration. [3]

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Candidate Name	Centre Number	Candidate Number

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INSERT for Questions 4 and 6

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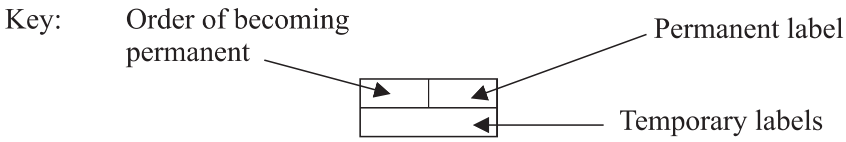
INSTRUCTIONS TO CANDIDATES

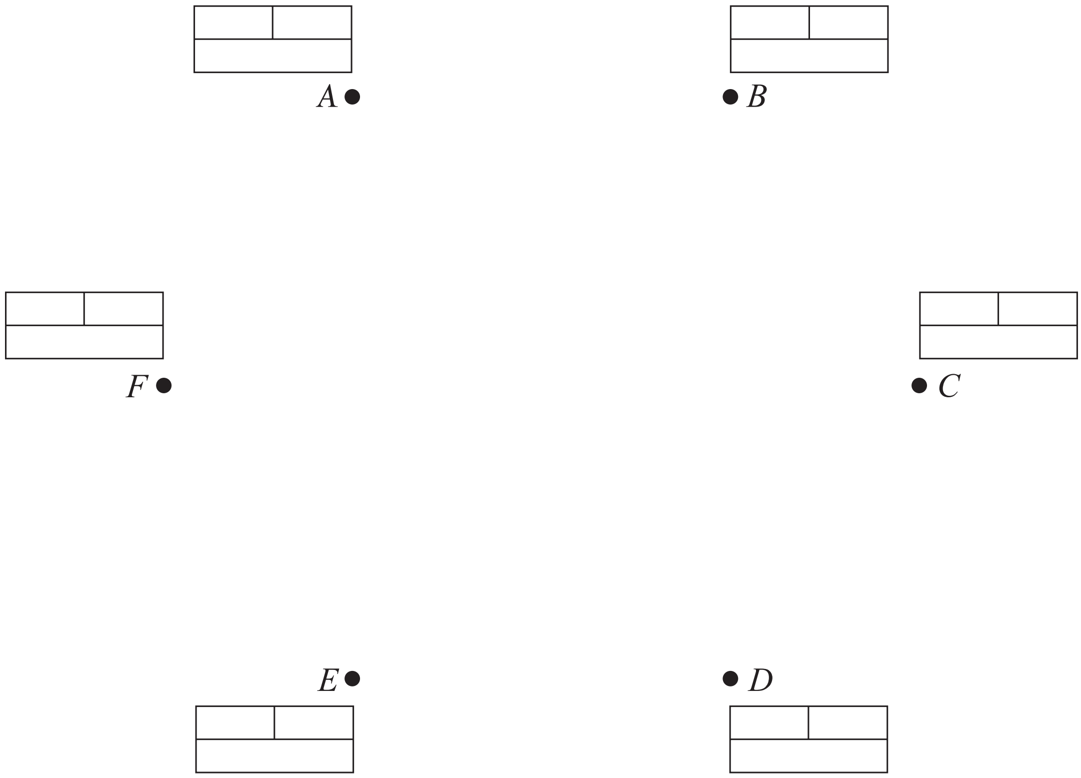
- This insert should be used to answer Questions **4** and **6**.
- Write your Name, Centre Number and Candidate Number in the spaces provided at the top of this page.
- Write your answers to Questions **4** and **6** in the spaces provided in this insert, and attach it to your answer booklet.

This insert consists of 3 printed pages and 1 blank page.

4 **(i)** and **(ii)**



Do not cross out your working values (temporary labels)



Shortest route from A to E = ...

(iii) ....

....

....

....

....

....

Length of shortest route = ... km

6 **(i)**

N	*D*	**Output**
30		
31		
32		
33		
34		
35		
36		
37		

(ii) ....

....

(iii) ....

(iv) ....

....

....

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