

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

18 JANUARY 2005

Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MATHEMATICS

Probability & Statistics 2

Tuesday

Afternoon

1 hour 20 minutes

2642

Additional materials: Answer booklet Graph paper List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer all the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- . The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 60.
- . Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- You are reminded of the need for clear presentation in your answers.

- 1 A secretary is typing a document. The number of typing mistakes the secretary makes per page can be modelled by a Poisson distribution with mean 2. Find the probability that
 - (i) in a document consisting of one page, the secretary makes more than 3 mistakes, [2]
 - (ii) in a document consisting of two pages, the secretary makes a total of fewer than 6 mistakes. [2]
- 2 The editor of a local newspaper is attempting to determine what proportion of the adults in the area served by the newspaper is interested in environmental matters. One issue of the newspaper therefore contains a questionnaire which readers are invited to complete and return.
 - (i) Give two reasons why the results obtained may be biased. [2]
 - (ii) Describe briefly an unbiased method of obtaining the information. [3]
- 3 The lifetime, T months, of properly made tap washers is modelled by a normal distribution with mean μ and standard deviation σ .
 - (i) It is given that P(T > 80.0) = 0.05 and P(T < 70.0) = 0.75. Find the values of μ and σ . [6]
 - (ii) Some tap washers are badly made and therefore have a very short lifetime. Give a reason why a normal distribution may not be a good model for the distribution of the lifetimes of all washers.
 [1]
- 4 A local government spokesman claims that at least three-quarters of the residents of a town are in favour of plans to build a new by-pass for the town. An opinion poll showed that 10 out of a random sample of 16 residents of the town were in favour of the plans. Test, at the 10% significance level, whether the results of the opinion poll are consistent with the spokesman's claim, stating your hypotheses clearly.
 [7]
- 5 The random variable *Y* has a Poisson distribution with mean 20.
 - (i) Use the exact distribution to calculate P(Y = 22). [2]
 - (ii) Use a suitable approximation to calculate P(Y = 22). [4]

[2]

[1]

- (iii) Calculate the percentage error in the approximate value.
- 6 (i) Explain what is meant by a Type I error.
 - (ii) The continuous random variable X has the distribution $N(\mu, \sigma^2)$. A test of the hypothesis $H_0: \mu = 25$ is carried out at the 5% significance level, once a day for 300 days. Given that on each day the value of μ is 25, use a normal approximation to find the probability that a Type I error is made on at least 20 days. [6]
 - (iii) Explain whether, in answering part (ii), it is necessary to assume that the outcomes of the tests are independent. [1]

7 Two models are proposed for the continuous random variable *X*. Model 1 has probability density function



The diagram shows the graph of $y = f_1(x)$.

(i) Find the upper quartile of X (i.e., find the value q such that P(X < q) = 0.75) according to model 1. [4]

Model 2 has probability density function

8

$$f_2(x) = \begin{cases} k(4 - x^2) & 0 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

- (ii) The graph of $y = f_2(x)$ intersects the y-axis at the point (0, 4k). Copy the diagram showing the graph of $y = f_1(x)$. On your copy sketch the graph of $y = f_2(x)$, explaining how you can tell without doing any integration that 4k < 1. [4]
- (iii) State whether the value of q obtained from model 1 is greater than, equal to, or less than the value given by model 2. Use your diagram to justify your answer. [2]
- (i) A random variable X has the distribution $N(\mu, \sigma^2)$. The mean of a sample of 5 observations of X is denoted by \overline{X} . State the distribution of \overline{X} , giving the values of any parameters. [2]
 - (ii) A group of scientists is attempting to identify subatomic particles called *ocrons*. Ocrons have a mean path length of less than 42 cm. The path lengths of a random sample of five particles thought to be ocrons are measured, and the mean path length of the sample is found to be 36.6 cm. Path lengths are known to be normally distributed random variables with standard deviation 8 cm. Carry out a test, at the 10% significance level, of whether the population mean path length is less than 42 cm, stating your hypotheses clearly.
 - (iii) A second group of scientists carries out a test that is identical, except that they use a 5% significance level. If the mean observed path length of the particles is consistent with a population mean of less than 42 cm, the scientists will claim that the particles are ocrons. State what the use of this smaller significance level suggests about the intentions of the scientists in deciding whether or not to claim that the observed particles are ocrons. [2]

BLANK PAGE

4

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (OCR) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.