OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2634

Pure Mathematics 4

Monday **10 JANUARY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 Find the general solution of the differential equation

$$\frac{dy}{dx} - y = e^{3x},$$

giving your answer in the form $y = f(x)$. [4]

2 Find the first three non-zero terms in the Maclaurin series for $e^{-x} \sin 2x$. (You may quote standard Maclaurin series expansions from the List of Formulae.) [5]

3 Prove by induction that

$$1 \times 4 + 2 \times 5 + 3 \times 6 + \ldots + n(n+3) = \tfrac{1}{3}n(n+1)(n+5)$$

for all integers $n \geqslant 1$. [5]

4 **(i)** Given that $y = \cos^{-1} 2x$, for $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$, find $\frac{dy}{dx}$. [2]

(ii) Use the substitution $x = \frac{1}{2}\sin\theta$ to find $\displaystyle\int \frac{1}{\sqrt{(1-4x^2)}}\,dx$. [3]

(iii) Hence show that $\cos^{-1} 2x + \sin^{-1} 2x = a$, where a is a constant to be found. [3]

5 The equation of a curve in polar coordinates is

$$r = \sin 2\theta + \cos 2\theta.$$

(i) Verify that $r = 0$ when $\theta = \frac{3}{8}\pi$, and sketch the part of the curve for which $0 \leqslant \theta \leqslant \frac{3}{8}\pi$. [3]

(ii) Find the exact area of the region enclosed between this part of the curve and the line $\theta = 0$. [5]

6 You are given that $f(r) = \dfrac{4}{(r+1)(r+3)}$.

(i) Express $f(r)$ in partial fractions. [2]

(ii) Hence find $\displaystyle\sum_{r=1}^{n} f(r)$. (You need not express your answer as a single fraction.) [4]

(iii) Show that the series in part **(ii)** is convergent, and state its sum to infinity. [2]

7 **(i)** The complex number z is such that $z^2 = 1 + \mathrm{i}\sqrt{3}$. Find the two possible values of z in the form $a + \mathrm{i}b$, where a and b are exact real numbers. [5]

(ii) With the value of z from part **(i)** such that the real part of z is positive, show on an Argand diagram the points A and B representing z and z^2 respectively. [2]

(iii) Specify two transformations which together map the line segment OA to the line segment OB, where O is the origin. [4]

8 The equation of a curve C is $y = \dfrac{x^2}{(x + 2a)(x + a)}$, where a is a positive constant.

(i) Find the equations of the asymptotes of C. [3]

(ii) Show that y cannot take values such that $-8 < y < 0$. [5]

(iii) Find the coordinates of the point where C intersects one of the asymptotes. [3]

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