OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2633

Pure Mathematics 3

Friday **14 JANUARY 2005** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are permitted to use only a scientific calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 Given that $|x| < \frac{1}{2}$, expand $(1 + 2x)^{-2}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

2 The parametric equations of a curve are

$$x = \theta \cos \theta, \qquad y = \sin \theta.$$

Find the gradient of the curve at the point for which $\theta = \pi$. [5]

3 **(i)** Express $3 \cos \theta + \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $0 < \alpha < \frac{1}{2}\pi$, stating the exact values of R and $\tan \alpha$. [3]

(ii) Hence solve the equation

$$3 \cos \theta + \sin \theta = 2,$$

for $0 < \theta < 2\pi$. [4]

4 **(i)** Verify that $\dfrac{x^3}{x^2 + 1} = x - \dfrac{x}{x^2 + 1}$. [1]

(ii) Hence find $\displaystyle\int \frac{x^3}{x^2 + 1}\, dx$. [3]

(iii) Use integration by parts to find $\displaystyle\int x \ln(x^2 + 1)\, dx$. [3]

5 The line L passes through the points P and Q with position vectors $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix}$ respectively.

(i) Find the equation of L, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [2]

(ii) Show that the point S with position vector $\begin{pmatrix} 9 \\ 5 \\ -2 \end{pmatrix}$ lies on L, and find the ratio of the length of PS to the length of QS. [3]

(iii) Find the acute angle between L and a line with direction vector $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$, giving your answer correct to the nearest degree. [3]

6 **(i)** Find the general solution of the differential equation

$$\frac{dy}{dx} = \left(\frac{y}{x}\right)^2,$$

giving your answer in the form $y = f(x)$. [5]

(ii) For the particular solution in which $y = 1$ when $x = 2$, find the value of y when $x = 8$. [3]

7 **(i)** Show that the substitution $y = \sqrt{x}$ transforms $\int \frac{1}{x(1+\sqrt{x})}\,dx$ to $\int \frac{2}{y(1+y)}\,dy$. [3]

(ii) Hence, by using partial fractions, find the exact value of $\int_4^9 \frac{1}{x(1+\sqrt{x})}\,dx$. [6]

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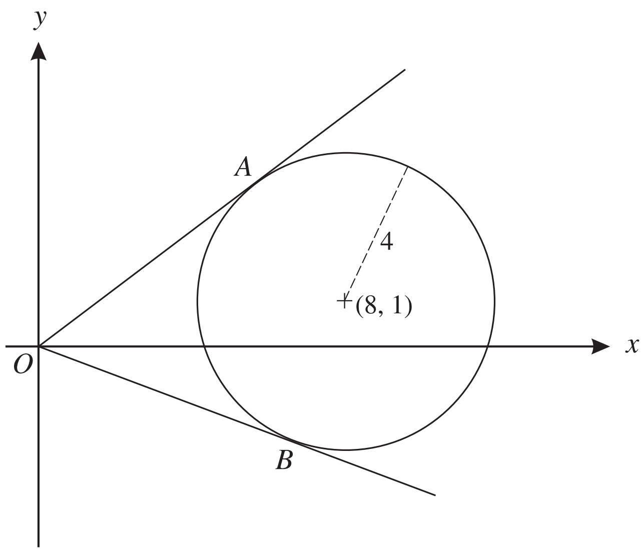

A circle has centre (8, 1) and radius 4. The points A and B on the circle are such that the tangents to the circle at A and B pass through the origin (see diagram).

(i) State the equation of the circle. [2]

(ii) The equation of any line through the origin is $y = mx$.

(a) Show that the x-coordinates of any points of intersection of this line and the circle are given by

$$x^2(1+m^2) - 2x(m+8) + 49 = 0.$$ [2]

(b) Hence or otherwise find the set of values of m for which the line meets the circle. [4]

(iii) Hence or otherwise prove that the exact value of $\tan AOB$ is $\frac{56}{33}$. [4]

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