GCE Examinations

Statistics
Module S2

Advanced Subsidiary / Advanced Level

Paper D

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. The continuous random variable X has the following cumulative distribution function:

$$\mathrm{F}(x) = \begin{cases} 0, & x < 2, \\ k(19x - x^2 - 34), & 2 \leq x \leq 5, \\ 1, & x > 5. \end{cases}$$

(a) Show that $k = \frac{1}{36}$. **(2 marks)**

(b) Find $\mathrm{P}(X > 4)$. **(2 marks)**

(c) Find and specify fully the probability density function $\mathrm{f}(x)$ of X. **(3 marks)**

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2. Suggest, with reasons, suitable distributions for modelling each of the following:

(a) the number of times the letter J occurs on each page of a magazine, **(3 marks)**

(b) the length of string left over after cutting as many 3 metre long pieces as possible from partly used balls of string,

(3 marks)

(c) the number of heads obtained when spinning a coin 15 times. **(3 marks)**

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3. A primary school teacher finds that exactly half of his year 6 class have mobile phones.

He decides to investigate whether the proportion of pupils with mobile phones is different from 0.5 in the year 5 class at his school. There are 25 pupils in the year 5 class.

(a) State the hypotheses that he should use. **(1 mark)**

(b) Find the largest critical region for this test such that the probability in each "tail" is less than 2.5%.

(4 marks)

(c) Determine the significance level of this test. **(1 mark)**

He finds that eight of the year 5 pupils have mobile phones and concludes that there is not sufficient evidence of the proportion being different from 0.5

(d) Stating the new hypotheses clearly, find if the number of year 5 pupils with mobile phones would have been significant if he had tested whether or not the proportion was less than 0.5 and used the largest critical region with a probability of less than 5%.

(3 marks)

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4. A hardware store is open on six days each week. On average the store sells 8 of a particular make of electric drill each week.

Find the probability that the store sells

(a) no more than 4 of the drills in a week, **(2 marks)**

(b) more than 2 of the drills in one day. **(5 marks)**

The store receives one delivery of drills at the same time each week.

(c) Find the number of drills that need to be in stock after a delivery for there to be at most a 5% chance of the store not having sufficient drills to meet demand before the next delivery.

(3 marks)

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5. In a party game, a bottle is spun and whoever it points to when it stops has to play next. The acute angle, in degrees, that the bottle makes with the side of the room is modelled by a rectangular distribution over the interval [0, 90].

Find the probability that on one spin this angle is

(a) between 25° and 38°, **(2 marks)**

(b) 45° to the nearest degree. **(2 marks)**

The bottle is spun ten times.

(c) Find the probability that the acute angle it makes with the side of the room is less than 10° more than twice.

(6 marks)

Turn over

6. A teacher is monitoring attendance at lessons in her department. She believes that the number of students absent from each lesson follows a Poisson distribution and wished to test the null hypothesis that the mean is 2.5 against the alternative hypothesis that it is greater than 2.5 She visits one lesson and decides on a critical region of 6 or more students absent.

(a) Find the significance level of this test. **(2 marks)**

(b) State any assumptions made in carrying out this test and comment on their validity.

(3 marks)

The teacher decides to undertake a wider study by looking at a sample of all the lessons that have taken place in the department during the previous four weeks.

(c) Suggest a suitable sampling frame. **(1 mark)**

She finds that there have been 96 pupils absent from the 30 lessons in her sample.

(d) Using a suitable approximation, test at the 5% level of significance the null hypothesis that the mean is 2.5 students absent per lesson against the alternative hypothesis that it is greater than 2.5. You may assume that the number of absences follows a Poisson distribution.

(6 marks)

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7. In a competition at a funfair, participants have to stay on a log being rotated in a pool of water for as long as possible. The length of time, in tens of seconds, that the competitors stay on the log is modelled by the random variable T with the following probability density function:

$$\mathrm{f}(t) = \begin{cases} k(t-3)^2, & 0 \le t \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Show that $k = \frac{1}{9}$. **(5 marks)**

(b) Sketch $\mathrm{f}(t)$ for all values of t. **(3 marks)**

(c) Show that the mean time that competitors stay on the log is 7.5 seconds. **(5 marks)**

When the competition is next run the organisers decide to make it easier at first by spinning the log more slowly and then increasing the speed of rotation. The length of time, in tens of seconds, that the competitors now stay on the log is modelled by the random variable S with the following probability density function:

$$\mathrm{f}(s) = \begin{cases} \frac{1}{12}(8-s^3), & 0 \le s \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

(d) Find the change in the mean time that competitors stay on the log. **(5 marks)**

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