GCE Examinations

Statistics
Module S2

Advanced Subsidiary / Advanced Level

Paper F

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 6 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Shaun Armstrong & Chris Huffer

1. *(a)* The random variable X follows a Poisson distribution with a mean of 1.4

Find $\mathrm{P}(X \leq 3)$. **(3 marks)**

(b) The random variable Y follows a binomial distribution such that $Y \sim \mathrm{B}(20, 0.6)$.

Find $\mathrm{P}(Y \leq 12)$. **(4 marks)**

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2. A driving instructor keeps records of all the learners she has taught. In order to analyse her success rate she wishes to take a random sample of 120 of these learners.

(a) Suggest a suitable sampling frame and identify the sampling units. **(2 marks)**

She believes that only 1 in 20 of the people she teaches fail to pass their test in their first two attempts. She decides to use her sample to test whether or not the proportion is different from this.

(b) Using a suitable approximation and stating clearly the hypotheses she should use, find the largest critical region for this test such that the probability in each "tail" is less than 2.5%.

(6 marks)

(c) State the significance level of this test. **(1 mark)**

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3. In an old computer game a white square representing a ball appears at random at the top of the playing area, which is 24 cm wide, and moves down the screen. The continuous random variable X represents the distance, in centimetres, of the dot from the left-hand edge of the screen when it appears. The distribution of X is rectangular over the interval [4, 28].

(a) Find the mean and variance of X. **(3 marks)**

(b) Find $\mathrm{P}(|X - 16| < 3)$. **(3 marks)**

During a single game, a player receives 12 "balls".

(c) Find the probability that the ball appears within 3 cm of the middle of the top edge of the playing area more than four times in a single game.

(3 marks)

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4. A music website is visited by an average of 30 different people per hour on a weekday evening. The site's designer believes that the number of visitors to the site per hour can be modelled by a Poisson distribution.

(a) State the conditions necessary for a Poisson distribution to be applicable and comment on their validity in this case.

(3 mark)

Assuming that the number of visitors does follow a Poisson distribution, find the probability that there will be

(b) less than two visitors in a 10-minute interval, **(3 marks)**

(c) at least ten visitors in a 15-minute interval. **(3 marks)**

(d) Using a suitable approximation, find the probability of the site being visited by more than 100 people between 6 pm and 9 pm on a Thursday evening.

(5 marks)

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5. Four coins are flipped together and the random variable H represents the number of heads obtained. Assuming that the coins are fair,

(a) suggest with reasons a suitable distribution for modelling H and give the value of any parameters needed,

(4 marks)

(b) show that the probability of obtaining more heads than tails is $\frac{5}{16}$. **(4 marks)**

The four coins are flipped 5 times and more heads are obtained than tails 4 times.

(c) Stating your hypotheses clearly, test at the 5% level of significance whether or not there is evidence of the probability of getting more heads than tails being more than $\frac{5}{16}$.

(5 marks)

Given that the four coins are all biased such that the chance of each one showing a head is 50% more than the chance of it showing a tail,

(d) find the probability of obtaining more heads than tails when the four coins are flipped together.

(4 marks)

Turn over

6. The continuous random variable X has the following probability density function:

$$f(x) = \begin{cases} \frac{1}{16}x, & 2 \le x \le 6, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Sketch f(x) for all values of x. **(2 marks)**

(b) Find E(X). **(3 marks)**

(c) Show that Var(X) = $\frac{11}{9}$. **(5 marks)**

(d) Define fully the cumulative distribution function F(x) of X. **(4 marks)**

(e) Show that the interquartile range of X is $2(\sqrt{7} - \sqrt{3})$. **(5 marks)**

END