1. (a) Explain the difference between a discrete and a continuous variable. **(2 marks)**

A random number generator on a calculator generates numbers, X, to 3 decimal places, in the range 0 to 1, e.g. 0·386. The variable X may be modelled by a continuous uniform distribution, having the probability density function $f(x)$, where

$$f(x) = 1 \qquad 0 < x < 1,$$
$$f(x) = 0 \qquad \text{otherwise.}$$

(b) Explain why this model is not totally accurate. **(1 mark)**

(c) Sketch the cumulative distribution function of X. **(2 marks)**

2. A video rental shop needs to find out whether or not videos have been rewound when they are returned; it will do this by taking a sample of returned videos

(a) State one advantage and one disadvantage of taking a sample. **(2 marks)**

(b) Suggest a suitable sampling frame. **(1 mark)**

(c) Describe the sampling units. **(1 mark)**

(d) Criticise the sampling method of looking at just one particular shelf of videos. **(2 marks)**

3. The random variable X is modelled by a binomial distribution $B(n, p)$, with $n = 20$ and p unknown. It is suspected that $p = 0{\cdot}4$.

(a) Find the critical region for the test of $H_0 : p = 0{\cdot}4$ against $H_1 : p \neq 0{\cdot}4$, at the 5% significance level. **(4 marks)**

(b) Find the critical region if, instead, the alternative hypothesis is $H_1 : p < 0{\cdot}4$. **(3 marks)**

4. A random variable X has the distribution B(80, 0·375).

(a) Write down the mean and variance of X. **(4 marks)**

(b) Use the Normal approximation to the binomial distribution to estimate $P(X > 40)$. **(7 marks)**

5. A traffic analyst is interested in the number of heavy lorries passing a certain junction. He counts the numbers of lorries in 100 five-minute intervals, and gets the following results:

Number of lorries in five-minute interval, X	0	1	2	3	4	5	6	7
Number of intervals	7	13	25	19	15	10	7	4

Q.5 continued on next page ...

5. continued ...

(a) Show that the mean of X is 3, and find the variance of X. **(4 marks)**

(b) Give two reasons for thinking that X can be modelled by a Poisson distribution. **(2 marks)**

After a new landfill site has been established nearby, a member of an environmental group notices that 18 lorries pass the junction in a period of 15 minutes. The group claims that this is evidence that the mean number of lorries per five-minute interval has increased.

(c) Test whether the group's claim is valid. Work at the 5% significance level, and state your hypotheses clearly. **(7 marks)**

6. In a particular parliamentary constituency, the percentage of Conservative voters at the last election was 35%, and the percentage who voted for the Monster Raving Loony party was 2%.

(a) Find the probability that a random sample of 10 electors includes at least two Conservative voters. **(3 marks)**

Use suitable approximations to find

(b) the probability that a random sample of 500 electors will include at least 200 who voted either Conservative or Monster Raving Loony, **(7 marks)**

(c) the probability that a random sample of 200 electors will have at least 5 Monster Raving Loony voters in it. **(4 marks)**

(d) One of (b) or (c) requires an adjustment to be made before a calculation is done. Explain what this adjustment is, and why it is necessary. **(2 marks)**

7. The fraction of sky covered by cloud is modelled by the random variable X with probability density function

$$\begin{aligned} f(x) &= 0 && x < 0, \\ f(x) &= kx^2(1-x) && 0 \le x \le 1, \\ f(x) &= 0 && x > 1. \end{aligned}$$

(a) Find k and sketch the graph of $f(x)$. **(4 marks)**

(b) Find the mean and the variance of X. **(6 marks)**

(c) Find the cumulative distribution function $F(x)$. **(4 marks)**

(d) Given that flying is prohibited when 85% of the sky is covered by cloud, show that cloud conditions allow flying nearly 90% of the time. **(3 marks)**