1. Explain what is meant by
   (a) a population, **(1 mark)**
   (b) a sampling unit. **(1 mark)**
   Suggest suitable sampling frames for surveys of
   (c) families who have holidays in Greece, **(1 mark)**
   (d) mothers with children under two years old. **(1 mark)**

2. A continuous random variable X has the probability density function

$$f(x) = k \qquad 5 \leq x \leq 15,$$
$$f(x) = 0 \qquad \text{otherwise}.$$

   (a) Find k and specify the cumulative density function $F(x)$. **(5 marks)**
   (b) Write down the value of $P(X < 8)$. **(1 mark)**

3. A coin is tossed 20 times, giving 16 heads.
   (a) Test at the 1% significance level whether the coin is fair, stating your hypotheses clearly. **(6 marks)**
   (b) Find the critical region for the same test at the 0·1% significance level. **(2 marks)**

4. Alison and Gemma play table tennis. Alison starts by serving for the first five points. The probability that she wins a point when serving is p.
   (a) Show that the probability that Alison is ahead at the end of her five serves is given by $p^3(6p^2 - 15p + 10)$. **(7 marks)**
   (b) Evaluate this probability when $p = 0{\cdot}6$. **(2 marks)**

5. In a certain school, 32% of Year 9 pupils are left-handed. A random sample of 10 Year 9 pupils is chosen.
   (a) Find the probability that none are left-handed. **(3 marks)**
   (b) Find the probability that at least two are left-handed. **(4 marks)**
   (c) Use a suitable approximation to find the probability of getting more than 5 but less than 15 left-handed pupils in a group of 35 randomly selected Year 9 pupils.
   Explain what adjustment is necessary when using this approximation. **(8 marks)**

6. A sample of radioactive material decays randomly, with an approximate mean of 1·5 counts per minute.
   (a) Name a distribution that would be suitable for modelling the number of counts per minute. Give any parameters required for the model. **(2 marks)**
   (b) Find the probability of at least 4 counts in a randomly chosen minute. **(3 marks)**
   (c) Find the probability of 3 counts or fewer in a random interval lasting 5 minutes. **(3 marks)**

   More careful measurements, over 50 one-minute intervals, give the following data for x, the number of counts per minute:

   $$\sum x = 84, \quad \sum x^2 = 226.$$

   (d) Decide whether these data support your answer to part (a). **(4 marks)**
   (e) Use the improved data to find probability of exactly two counts in a given one-minute interval. **(3 marks)**

7. Each day on the way to work, a commuter encounters a similar traffic jam. The length of time, in 10-minute units, spent waiting in the traffic jam is modelled by the random variable T with the cumulative distribution function:

   $$\begin{aligned} F(t) &= 0 && t < 0, \\ F(t) &= \frac{t^2(3t^2 - 16t + 24)}{16} && 0 \le t \le 2, \\ F(t) &= 1 && t > 2. \end{aligned}$$

   (a) Show that 0·77 is approximately the median value of T. **(3 marks)**
   (b) Given that he has already waited for 12 minutes, find the probability that he will have to wait another 3 minutes. **(5 marks)**
   (c) Find, and sketch, the probability density function of T. **(4 marks)**
   (d) Hence find the modal value of T. **(5 marks)**
   (e) Comment on the validity of this model. **(1 mark)**

