1. (a) Briefly describe the difference between a census and a sample survey.   **(1 mark)**

    (b) Illustrate the difference by considering the case of a village council which has to decide whether or not to build a new village hall.   **(2 marks)**

    Given that the council decides to use a sample survey,

    (c) suggest suitable sampling units.   **(1 mark)**

2. The number of copies of *The Statistician* that a newsagent sells each week is modelled by a Poisson distribution. On average, he sells 1·5 copies per week.

    (a) Find the probability that he sells no copies in a particular week.   **(1 mark)**

    (b) If he stocks 5 copies each week, find the probability he will not have enough copies to meet that week's demand.   **(2 marks)**

    (c) Find the minimum number of copies that he should stock in order to have at least a 95% probability of being able to satisfy the week's demand.   **(3 marks)**

3. A die is rolled 60 times, and results in 16 sixes.

    (a) Use a suitable approximation to test, at the 5% significance level, whether the probability of scoring a six is $\frac{1}{6}$ or not.   State your hypotheses clearly.   **(6 marks)**

    (b) Describe how you would change the test if you wished to investigate whether the probability of scoring a six is greater than $\frac{1}{6}$. Carry out this modified test.   **(4 marks)**

4. A continuous random variable X has the cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 4, \\ \frac{1}{84}(x^2 - 16) & 4 \le x \le 10, \\ 1 & x > 10. \end{cases}$$

    (a) Find the median value of X.   **(2 marks)**

    (b) Find the interquartile range for X.   **(4 marks)**

    (c) Find the probability density function $f(x)$ of X.   **(3 marks)**

    (d) Sketch the graph of $f(x)$ and hence write down the mode of X, explaining how you obtain your answer from the graph.   **(3 marks)**

5. Lupin seeds are sold in packets of 15. On average, 9 seeds in a packet are green and 6 are red. Find, to 2 decimal places, the probability that in any particular packet there are

   (a) less than 2 red seeds, **(3 marks)**

   (b) more red than green seeds. **(2 marks)**

   The seeds from 10 packets are then combined together.

   (c) Use a suitable approximation to find the probability that the total number of green seeds is more than 100. **(7 marks)**

6. Patients suffering from 'flu are treated with a drug. The number of days, t, that it then takes for them to recover is modelled by the continuous random variable T with the probability density function

$$f(t) = \frac{3t^2(4-t)}{64} \qquad 0 \le t \le 4,$$

$$f(t) = 0 \qquad \text{otherwise.}$$

   (a) Find the mean and standard deviation of T. **(7 marks)**

   (b) Find the probability that a patient takes more than 3 days to recover. **(4 marks)**

   (c) Two patients are selected at random. Find the probability that they both recover within three days. **(2 marks)**

   (d) Comment on the suitability of the model. **(1 mark)**

7. In a certain field, daisies are randomly distributed, at an average density of 0·8 daisies per cm². One particular patch, of area 1 cm², is selected at random.

   Assuming that the number of daisies per cm² has a Poisson distribution,

   (a) find the probability that the chosen patch contains

       (i) no daisies, **(1 mark)**

       (ii) one daisy. **(1 mark)**

   Ten such patches are chosen. Using your answers to part (a),

   (b) find the probability that the total number of daisies is less than two. **(4 marks)**

   (c) By considering the distribution of daisies over patches of 10 cm², use the Poisson distribution to find the probability that a particular area of 10 cm² contains no more than one daisy. **(3 marks)**

   (d) Compare your answers to parts (b) and (c). **(2 marks)**

   (e) Use a suitable approximation to find the probability that a patch of area 1 m² contains more than 8 100 daisies. **(6 marks)**