

STATISTICS 2 (A) TEST PAPER 6 : ANSWERS AND MARK SCHEME

1.	(a) List of all drivers insured with that company	B1	
	(b) Individual drivers	(c) Sample is quicker and cheaper	B1; B2 4
2.	(a) Continuous uniform distribution on [0, 360]	B1	
	(b) Mean = 180	s.d. = $\sqrt{(360^2 \div 12)} = \sqrt{10800} = 103.9$	B1 M1 A1 4
3.	(a) Mean = $322/150 = 2.15$, variance = $982/150 - 2 \cdot 147^2 = 1.94$	M1 A1 M1 A1	
	(b) Mean \approx Variance, which suggests Poisson	B1	
	(c) Assuming $\lambda = 2.14666$, no. of errors in 6 pages is Po(12.88)	B1 B1	
	then $P(X \geq 15) > P(X = 15) = 0.0868 > 5\%$, so do not reject H_0	M1 A1 A1	10
4.	(a) $X \sim B(5, 0.2)$	$P(X = 0) = 0.8^5 = 0.3277$	M1 A1
	(b) $P(X > 2) = 1 - P(X \leq 2) = 1 - 0.9421 = 0.0579$ (from tables)	M1 A1 A1	
	(c) No of lates in 7 weeks is distributed $B(35, 0.2) \approx N(7, 5.6)$	M1 A1	
	$P(X > 10) = P(X > 10.5) = P(Z > 3.5/\sqrt{5.6}) = P(Z > 1.48)$	M1 A1 A1	
	$= 1 - 0.9306 = 0.0694$	M1 A1	12
5.	(a) $X \sim Po(4)$, so $P(X < 2) = 0.0916$	B1 M1 A1	
	(b) $P(3 < X < 10) = 0.9919 - 0.4335 = 0.558$	M1 M1 A1	
	(c) H_0 : mean number of bubbles is still 0.1 per cm^3 ; H_1 : mean < 0.1	B1 B1	
	Under H_0 , no. of bubbles in $60 cm^3$ is Po(6)	B1	
	Then $P(X \leq 1) = 0.0174$, so do not reject H_0 at 1% level	M1 A1 A1	12
6.	(a) $f'(x) = \frac{4}{27}(6x - 3x^2) = 0$ when $x = 2$, so mode = 2	M1 A1 A1	
	(b) $E(x) = \frac{4}{27} \int_0^3 3x^3 - x^4 dx = \frac{4}{27} \left[\frac{3x^4}{4} - \frac{x^5}{5} \right]_0^3 = 1.8$	M1 A1 A1	
	(c) $F(x) = 0$ ($x < 0$), $F(x) = \frac{4}{27} \left(x^3 - \frac{x^4}{4} \right)$ ($0 \leq x \leq 3$), $F(x) = 1$ ($x > 3$)	B1 M1 A1 B1	
	(d) For median m , $F(m) = 0.5$ Now $F(1.84) = 0.498 < 0.5$ and $F(1.85) = 0.504 > 0.5$, so $1.84 < m < 1.85$	M1 A1 A1 A1	
	(e) Mean $<$ median $<$ mode Negative skew	B1	15
7.	(a) $\int_2^{10} f(x) dx = 1$, so $k \int_2^{10} (-x^2 + 12x - 20) dx = 1$	M1 M1 A1	
	$k \left[-\frac{1}{3}x^3 + 6x^2 - 20x \right]_2^{10} = 1$ $\frac{256k}{3} = 1$ $k = \frac{3}{256}$	M1 A1	
	By symmetry of parabola, mean = 6	B1	
	(b) $E(X^2) = \frac{3}{256} \int_2^{10} (-x^4 + 12x^3 - 20x^2) dx$	M1 A1	
	$= \frac{3}{256} \left[-\frac{x^5}{5} + 3x^4 - \frac{20x^3}{3} \right]_2^{10} = 39.2$ $Var(X) = 3.2$	M1 A1 A1	
	Standard deviation = $\sqrt{3.2} = 1.789$, i.e. £1789	A1	
	(c) $P(X > 8) = \int_8^{10} f(x) dx = \frac{3}{256} \left[-\frac{1}{3}x^3 + 6x^2 - 20x \right]_8^{10} = 0.156$	M1 A1 M1 A1	
	(d) $0.156^4 = 0.000592$	M1 A1	18