GCE Examinations

Advanced Subsidiary / Advanced Level

Statistics
Module S2

Paper D

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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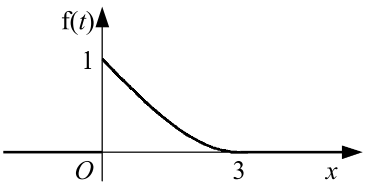
S2 Paper D – Marking Guide

1.	*(a)*	F(5) = 1	M1	
		$k(95 - 25 - 34) = 1$; $36k = 1$ $\therefore k = \frac{1}{36}$	A1	
	(b)	$P(X > 4) = 1 - F(4)$	M1	
		$= 1 - \frac{1}{36}(76 - 16 - 34) = \frac{5}{18}$	A1	
	(c)	$f(x) = F'(x) = \frac{1}{36}(19 - 2x)$	M1 A1	
		$\therefore f(x) = \begin{cases} \frac{1}{36}(19-2x), & 2 \le x \le 5, \\ 0, & \text{otherwise.} \end{cases}$	A1	**(7)**
2.	*(a)*	Poisson	B1	
		e.g. J occurs singly, at random, at constant rate	B2	
	(b)	continuous uniform	B1	
		e.g. initial lengths random $\therefore$ equal chance of any length 0 to 3 left over	B2	
	(c)	binomial	B1	
		e.g. fixed no. of spins, two outcomes, fixed prob. of head	B2	**(9)**
3.	*(a)*	$H_0 : p = \frac{1}{2}$ $\quad H_1 : p \neq \frac{1}{2}$	B1	
	(b)	let X = no. with mobile phones $\therefore X \sim B(25, \frac{1}{2})$	M1	
		$P(X \le 7) = 0.0216$; $P(X \le 17) = 0.9784$	M1 A1	
		$\therefore$ C.R. is $X \le 7$ or $X \ge 18$	A1	
	(c)	$0.0216 + 0.0216 = 0.0432$	A1	
	(d)	$H_0 : p = \frac{1}{2}$ $\quad H_1 : p < \frac{1}{2}$	B1	
		$P(X \le 8) = 0.0539$	M1	
		more than 5% $\therefore$ not significant	A1	**(9)**
4.	*(a)*	let X = no. of sales per week $\therefore X \sim Po(8)$	M1	
		$P(X \le 4) = 0.0996$	A1	
	(b)	let Y = no. of sales per day $\therefore Y \sim Po(\frac{4}{3})$	M1	
		$P(Y > 2) = 1 - P(Y \le 2)$	M1	
		$= 1 - e^{-\frac{4}{3}}(1 + \frac{4}{3} + \frac{(\frac{4}{3})^2}{2})$	M1 A1	
		$= 1 - 0.8494 = 0.1506$ (4sf)	A1	
	(c)	$P(X \le 12) = 0.9362$; $P(X \le 13) = 0.9658$	M1 A1	
		$\therefore$ need 13 in stock	A1	**(10)**

5.	*(a)*	$13 \times \frac{1}{90} = \frac{13}{90}$ or 0.1444 (4sf)	M1 A1	
	(b)	P(44.5° to 45.5°) $\therefore \frac{1}{90}$	M1 A1	
	(c)	$P(< 10°) = 10 \times \frac{1}{90} = \frac{1}{9}$	A1	
		let X = no. of times < 10° $\therefore X \sim B(10, \frac{1}{9})$	M1	
		$P(X > 2) = 1 - P(X \leq 2)$	M1	
		$= 1 - [(\frac{8}{9})^{10} + 10(\frac{1}{9})(\frac{8}{9})^9 + \frac{10\times9}{2}(\frac{1}{9})^2(\frac{8}{9})^8]$	M1 A1	
		$= 1 - 0.9094 = 0.0906$ (3sf)	A1	**(10)**
6.	*(a)*	let X = no. absent per lesson $\therefore X \sim Po(2.5)$		
		$P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.9580 = 0.0420$	M1 A1	
	(b)	assumes absences occur independently and at constant rate ill students may infect others and rate may vary at different times of year but assumptions fairly reasonable	B3	
	(c)	registers for all classes	B1	
	(d)	let Y = no. absent per 30 lessons $\therefore Y \sim Po(75)$	M1	
		use N approx. $A \sim N(75, 75)$	M1	
		$P(Y \geq 96) \approx P(A > 95.5)$	M1	
		$= P(Z > \frac{95.5 - 75}{\sqrt{75}}) = P(Z > 2.367)$	A1	
		$= 1 - 0.9909 = 0.0091$	A1	
		less than 5% $\therefore$ significant, there is evidence of more absent per lesson	A1	**(12)**
7.	*(a)*	$\int_0^3 k(t-3)^2 \; dt = 1$	M1	
		$k\int_0^3 t^2 - 6t + 9 \; dt = 1$	M1	
		$\therefore k[\frac{1}{3}t^3 - 3t^2 + 9t]_0^3 = 1$	A1	
		$\therefore k[(9 - 27 + 27) - (0)] = 1;\; 9k = 1;\; k = \frac{1}{9}$	M1 A1	
	(b)		B3	
	(c)	$E(T) = \int_0^3 t \times \frac{1}{9}(t-3)^2 \; dt = \frac{1}{9}\int_0^3 t^3 - 6t^2 + 9t \; dt$	M1	
		$= \frac{1}{9}[\frac{1}{4}t^4 - 2t^3 + \frac{9}{2}t^2]_0^3$	A1	
		$= \frac{1}{9}[(\frac{81}{4} - 54 + \frac{81}{2}) - (0)] = \frac{3}{4}$	M1 A1	
		$\therefore$ mean time $= \frac{3}{4} \times 10 = 7.5$ s	A1	
	(d)	$E(S) = \int_0^2 s \times \frac{1}{12}(8 - s^3) \; ds = \frac{1}{12}\int_0^2 8s - s^4 \; ds$	M1	
		$= \frac{1}{12}[4s^2 + \frac{1}{5}s^5]_0^2$	A1	
		$= \frac{1}{12}[(16 - \frac{32}{5}) - (0)] = \frac{4}{5}$	M1 A1	
		$\therefore$ new mean $= \frac{4}{5} \times 10 = 8$ s $\therefore$ increased by 0.5 s	A1	**(18)**
			Total	**(75)**

Performance Record – S2 Paper D

Question no.	1	2	3	4	5	6	7	Total
Topic(s)	c.d.f., p.d.f.	modelling	binomial, hyp. test	Poisson	rect. dist., binomial	Poisson, hyp. test., sampling, N approx.	p.d.f., mean	
Marks	7	9	9	10	10	12	18	75
Student								