1. An insurance company is investigating how often its customers crash their cars.
   (a) Suggest an appropriate sampling frame. **(1 mark)**
   (b) Describe the sampling units. **(1 mark)**
   (c) State the advantage of a sample survey over a census in this case. **(2 marks)**

2. A searchlight is rotating in a horizontal circle. It is assumed that that, at any moment, the centre of its beam is equally likely to be pointing in any direction. The random variable X represents this direction, expressed as a bearing in the range 000° to 360°.
   (a) Specify a suitable model for the distribution of X. **(1 mark)**
   (b) Find the mean and the standard deviation of X. **(3 marks)**

3. A secretarial agency carefully assesses the work of a new recruit, with the following results after 150 pages:

No of errors	0	1	2	3	4	5	6
No of pages	16	38	41	29	17	7	2

   (a) Find the mean and variance of the number of errors per page. **(4 marks)**
   (b) Explain how these results support the idea that the number of errors per page follows a Poisson distribution. **(1 mark)**
   (c) After two weeks at the agency, the secretary types a fresh piece of work, six pages long, which is found to contain 15 errors.
   The director suspects that the secretary was trying especially hard during the early period and that she is now less conscientious. Using a Poisson distribution with the mean found in part (a), test this hypothesis at the 5% significance level. **(5 marks)**

4. A certain Sixth Former is late for school once a week, on average. In a particular week of 5 days, find the probability that
   (a) he is not late at all, **(2 marks)**
   (b) he is late more than twice. **(3 marks)**

   In a half term of seven weeks, lateness on more than ten occasions results in loss of privileges the following half term.
   (c) Use the Normal approximation to estimate the probability that he loses his privileges. **(7 marks)**

5. A certain type of steel is produced in a foundry. It has flaws (small bubbles) randomly distributed, and these can be detected by X-ray analysis. On average, there are 0·1 bubbles per cm^3, and the number of bubbles per cm^3 has a Poisson distribution.

In an ingot of 40 cm^3, find

(a) the probability that there are less than two bubbles, **(3 marks)**

(b) the probability that there are more than 3 but less than 10 bubbles. **(3 marks)**

A new machine is being considered. Its manufacturer claims that it produces fewer bubbles per cm^3. In a sample ingot of 60 cm^3, there is just one bubble.

(c) Carry out a hypothesis test at the 1% significance level to decide whether the new machine is better. State your hypotheses and conclusion carefully. **(6 marks)**

6. A random variable X has a probability density function given by

$$f(x) = \frac{4x^2(3-x)}{27} \qquad 0 \le x \le 3,$$
$$f(x) = 0 \qquad \text{otherwise.}$$

(a) Find the mode of X. **(3 marks)**

(b) Find the mean of X. **(3 marks)**

(c) Specify completely the cumulative distribution function of X. **(4 marks)**

(d) Deduce that the median, m, of X satisfies the equation $m^4 - 4m^3 + 13{\cdot}5 = 0$, and hence show that $1{\cdot}84 < m < 1{\cdot}85$. **(4 marks)**

(e) What do these results suggest about the skewness of the distribution? **(1 mark)**

7. A corner-shop has weekly sales (in thousands of pounds), which can be modelled by the continuous random variable X with probability density function

$$f(x) = k(x-2)(10-x) \qquad 2 \le x \le 10,$$
$$f(x) = 0 \qquad \text{otherwise.}$$

(a) Show that $k = \frac{3}{256}$ and write down the mean of X. **(6 marks)**

(b) Find the standard deviation of the weekly sales. **(6 marks)**

(c) Find the probability that the sales exceed £8 000 in any particular week. **(4 marks)**

If the sales exceed £8 000 per week for 4 consecutive weeks, the manager gets a bonus.

(d) Find the probability that the manager gets a bonus in February. **(2 marks)**