1. (a) Explain briefly why it is often useful to take a sample from a population. **(2 marks)**

   (b) Suggest a suitable sampling frame for a local council to use to survey attitudes towards a proposed new shopping centre. **(1 mark)**

2. A certain type of lettuce seed has a 12% failure rate for germination. In a new sample of 25 genetically modified seeds, only 1 did not germinate.

   Clearly stating your hypotheses, test, at the 1% significance level, whether the GM seeds are better. **(6 marks)**

3. A random variable X has a Poisson distribution with a mean, λ, which is assumed to equal 5.

   (a) Find $P(X = 0)$. **(1 mark)**

   (b) In 100 measurements, the value 0 occurs three times. Find the highest significance level at which you should reject the original hypothesis in favour of $\lambda < 5$. **(8 marks)**

4. The waiting time, in minutes, at a dentist is modelled by the continuous random variable T with probability density function

$$f(t) = k(10 - t) \qquad 0 \leq t \leq 10,$$
$$f(t) = 0 \qquad \text{otherwise.}$$

   (a) Sketch the graph of $f(t)$ and find the value of k. **(4 marks)**

   (b) Find the mean value of T. **(4 marks)**

   (c) Find the 95th percentile of T. **(3 marks)**

   (d) State whether you consider this function to be a sensible model for T and suggest how it could be modified to provide a better model. **(2 marks)**

5. A textbook contains, on average, 1·2 misprints per page. Assuming that the misprints are randomly distributed throughout the book,

   (a) specify a suitable model for X, the random variable representing the number of misprints on a given page. **(1 mark)**

   (b) Find the probability that a particular page has more than 2 misprints. **(3 marks)**

   (c) Find the probability that Chapter 1, with 8 pages, has no misprints at all. **(2 marks)**

   Chapter 2 is longer, at 20 pages.

   (d) Use a suitable approximation to find the probability that Chapter 2 has less than ten misprints altogether. Explain what adjustment is necessary when making this approximation. **(7 marks)**

6. On a production line, bags are filled with cement and weighed as they emerge. It is found that 20% of the bags are underweight. In a random sample consisting of n bags, the variance of the number of underweight bags is found to be 2·4.

(a) Show that $n = 15$. **(2 marks)**

(b) Use cumulative binomial probability tables to find the probability that, in a further random sample of 15 bags, the number that are underweight is

(i) less than 3, **(3 marks)**

(ii) at least 5. **(2 marks)**

Ten samples of 15 bags each are tested. Find the probability that

(c) all these batches contain less than 5 underweight bags, **(3 marks)**

(d) the fourth batch tested is the first to contain less than 5 underweight bags. **(3 marks)**

7. A continuous random variable X has a probability density function given by

$$f(x) = \frac{x^2}{312} \qquad 4 \leq x \leq 10,$$

$$f(x) = 0 \qquad \text{otherwise.}$$

(a) Find $E(X)$. **(3 marks)**

(b) Find the variance of X. **(4 marks)**

(c) Find the cumulative distribution function $F(x)$, for all values of x. **(5 marks)**

(d) Hence find the median value of X. **(3 marks)**

(e) Write down the modal value of X. **(1 mark)**

It is sometimes suggested that, for most distributions,

$$2 \times (\text{median} - \text{mean}) \approx \text{mode} - \text{median}.$$

(f) Show that this result is not satisfied in this case, and suggest a reason why. **(2 marks)**

