

**STATISTICS 2 (A) TEST PAPER 5 : ANSWERS AND MARK SCHEME**

1. (a) Quicker and cheaper than a census B2  
 (b) Use the electoral roll B1 3
2.  $X \sim B(25, p)$   $H_0 : p = 0.12$   $H_1 : p < 0.12$  B1 B1  
 Under  $H_0$ ,  $P(X = 0 \text{ or } 1) = 0.88^{25} + 25 \times 0.88^{24} \times 0.12 = 0.181 > 1\%$  M1 M1 A1  
 Therefore, no significant evidence in favour of  $H_1$  at the 1% level A1 6
3. (a) If mean = 5,  $X \sim \text{Po}(5)$   $P(X = 0) = 0.0067$  B1  
 (b)  $X \sim \text{Po}(\lambda)$   $H_0 : \lambda = 5$   $H_1 : \lambda < 5$  B1 B1  
 Under  $H_0$ , no. of '0's in 100 measurements  $\sim \text{Po}(0.67)$  M1 A1  
 $P(X \geq 3) = 1 - e^{-0.67}(1 + 0.67 + 0.67^2/2!) = 0.031 = 3.1\%$  M1 A1 A1  
 Reject  $H_0$  at the 5% significance level, but not at the 1% level. A1 9
4. (a) Graph : straight line from (0, 10k) to (10, 0); on x-axis otherwise B2  
 $\frac{1}{2} \times 10 \times 10k = 1$   $k = \frac{1}{50}$  M1 A1
- (b)  $E(T) = \int_0^{10} t f(t) dt = \frac{1}{50} \int_0^{10} 10t - t^2 dt = \frac{1}{50} \left[ 5t^2 - \frac{t^3}{3} \right]_0^{10} = 3\frac{1}{3}$  M1 A1 M1 A1
- (c) From graph,  $\frac{1}{2}(10 - p) \left( \frac{10 - p}{50} \right) = \frac{5}{100}$   $(10 - p)^2 = 5$   $p = 7.76$  M1 A1 A1
- (d) Some wait more than 10 mins; more gradual slope needed B1 B1 13
5. (a) Poisson :  $X \sim \text{Po}(1.2)$  B1  
 (b)  $P(X > 2) = 1 - e^{-1.2} - 1.2e^{-1.2} - 1.2^2 e^{-1.2}/2! = 0.121$  M1 A1 A1  
 (c)  $P(X = 0) = e^{-1.2} = 0.301$   $P(0 \text{ in Ch. 1}) = 0.301^8 = 0.0000677$  M1 A1  
 (d) Total for Ch. 2  $\sim \text{Po}(24) \approx N(24, 24)$  Then  $P(X < 10)$  M1 A1  
 $= P(X < 9.5) = P(Z < -14.5/4.90) = P(Z < -2.96) = 0.0015$  M1 A1 M1 A1  
 Continuity correction needed, from discrete to continuous B1 13
6. (a)  $B(n, 0.2) : \text{Var} = n(0.2)(0.8) = 2.4$   $n = 15$  M1 A1  
 (b)  $X \sim B(15, 0.2)$  (i)  $P(X < 3) = P(X \leq 2) = 0.398$  B1 M1 A1  
 (ii)  $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.8358 = 0.164$  M1 A1  
 (c)  $X \sim B(10, 0.8358) : P(X = 10) = 0.8358^{10} = 0.166$  B1 M1 A1  
 (d)  $0.1642^3 \times 0.8358 = 0.00370$  M1 A1 A1 13
7. (a)  $E(X) = \int_4^{10} \frac{x^3}{312} dx = \left[ \frac{x^4}{1248} \right]_4^{10} = 7.81$  M1 A1 A1  
 (b)  $\text{Var}(X) = \int_4^{10} \frac{x^4}{312} dx - 7.81^2 = \left[ \frac{x^5}{1560} \right]_4^{10} - 7.808^2 = 2.49$  M1 A1 M1 A1  
 (c)  $F(x) = 0$  ( $x < 4$ ),  $F(x) = \int_4^x \frac{u^2}{312} du = \frac{x^3 - 64}{936}$  ( $4 \leq x \leq 10$ ), B1 M1 A1 A1  
 $F(x) = 1$  ( $x > 10$ ) B1
- (d) For median  $m$ ,  $\frac{m^3 - 64}{936} = \frac{1}{2}$   $m^3 = 532$   $m = 8.10$  M1 A1 A1  
 (e) By inspection, mode = 10 B1  
 (f)  $2(8.1 - 7.81) = 0.58$   $10 - 8.1 = 1.9$  Not very B1  
 similar; mode is an extreme point and is not centrally located B1 18