STATISTICS 2 (A) TEST PAPER 2 : ANSWERS AND MARK SCHEME

1.	(a) Quicker / cheaper to carry out; do not destroy whole population	B2	
	(b) A list of the population (c) A list of residents of the road	B1 B1	4
2.	(a) $X \sim \text{Po}(\lambda)$ $\text{H}_0 : \lambda = 3{\cdot}5,\ \text{H}_1 : \lambda > 3{\cdot}5.$	B1 B1	
	Under H_0, $\text{P}(X \geq 7) = 1 - 0{\cdot}9347 > 5\%$, so do not reject H_0	M1 A1 A1	
	(b) Need $\text{P}(X \geq n) < 0{\cdot}001$ $\text{P}(X \leq n-1) > 0{\cdot}999$	M1 A1	
	$n - 1 = 11$, so 12 calls are required	A1	8
3.	(a) $X \sim \text{B}(20, 0.35)$ From tables, $\text{P}(X \leq 4) = 0{\cdot}118$	M1 A1	
	(b) $\text{P}(X \leq 8) - \text{P}(X \leq 7) = 0{\cdot}7624 - 0{\cdot}6010 = 0{\cdot}161$	M1 A1	
	(c) $\text{B}(100, 0{\cdot}35) \approx \text{N}(35, 22{\cdot}75)$ $\text{P}(X < 25) = \text{P}(X < 24{\cdot}5)$	M1 A1 M1	
	$= \text{P}(Z < -10{\cdot}5/4{\cdot}77) = \text{P}(Z < -2{\cdot}201) = 1 - 0{\cdot}9861 = 0{\cdot}0139$	M1 A1 A1	10
4	(a) Graph drawn through (0, 0) and (3, 0) (b) 150 hours	B2; B1	
	(c) $\text{P}(X < 2) = \int_0^2 \text{f}(x)\,\text{d}x = \frac{2}{9}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^2 = \frac{2}{9}\left[6 - \frac{8}{3}\right] = 0{\cdot}741$	M1 A1	
	$\text{P}(X > 2{\cdot}5) = \text{P}(X < 0{\cdot}5) = \frac{2}{9}\left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^{0.5} = \frac{2}{9}\left[\frac{3}{8} - \frac{1}{24}\right] = 0{\cdot}0741$	M1 A1 A1	
	(d) $0{\cdot}0741 \div (1 - 0{\cdot}741) = 0{\cdot}286$	M1 A1	
	(e) Too rigid a cut-off; some bulbs might last longer than 300 hours	B1	11
5.	(a) Mean $= 92/40 = 2.3$ and variance $= 300/40 - 2{\cdot}3^2 = 2{\cdot}21$	M1 A1 M1 A1	
	(b) Poisson, because mean $\approx$ variance, and data positively skewed	B1 B1	
	(c) If mean $= 2{\cdot}3$, $\text{P}(X \geq 2) = 1 - \text{e}^{-2\cdot3} - 2{\cdot}3\text{e}^{-2\cdot3} = 0{\cdot}669$	M1 A1	
	If mean $= 1{\cdot}9$, $\text{P}(X \geq 2) = 1 - \text{e}^{-1\cdot9} - 1{\cdot}9\text{e}^{-1\cdot9} = 0{\cdot}566$	M1 A1	
	More likely to get at least 2 currants with the first machine	A1	11
6.	(a) $X \sim \text{B}(10, p)$; $\text{H}_0 : p = 0{\cdot}05$, $\text{H}_1 : p > 0{\cdot}05$	B1 B1	
	Under H_0, $\text{P}(X \geq 2) = 1 - 0{\cdot}9139 = 0{\cdot}0861 > 1\%$, so accept H_0	M1 A1 A1	
	(b) Assumed that apples are selected randomly	B1	
	(c) Now have $\text{B}(60, 0{\cdot}05)$, assuming H_0. This is approx. $\text{Po}(3)$	M1 A1	
	$\text{P}(X \geq 10) = 1 - 0{\cdot}9989 = 0{\cdot}0011 < 1\%$, so reject H_0	M1 A1 A1	
	(d) More data gives greater evidence and can be more decisive	B1	12
7.	(a) Mean = 5 by symmetry Standard dev. $= \sqrt{(100/12)} = 2{\cdot}89$	B1 M1 A1	
	(b) Mean = 5 (c) For area 1 under pdf, $\frac{1}{2} \times 10 \times 5c = 1$, so $c = \frac{1}{25}$	B1; M1 A1	
	Variance $= \int_0^{10} \text{f}(x)\,\text{d}x - 5^2 = \frac{1}{25}\left(\left[\frac{x^4}{4}\right]_0^5 + \left[\frac{10x^3}{3} - \frac{x^4}{4}\right]_5^{10}\right) - 25 = 4\frac{1}{6}$	M1 A1 M1 A1	
	so s.d. $= 2{\cdot}04$	A1	
	(d) $\text{P}(4 < X < 6) = 2 \times \frac{1}{2} \times 1 \times \left(\frac{4}{25} + \frac{5}{25}\right) = \frac{9}{25}$ or $0{\cdot}36$	M1 A1 A1	
	(e) $\text{P}(4 < X < 6) = \text{P}(-1/2{\cdot}04 < Z < 1/2{\cdot}04) = \text{P}(-0{\cdot}49 < Z < 0{\cdot}49)$	M1 M1 A1	
	$= 2(0{\cdot}1879) = 0{\cdot}376$	A1	
	(f) Similar; slightly more concentration near the mean for the Normal model. People are aiming at the middle, so this is probably better	B1	19