STATISTICS 2 (A) TEST PAPER 10 : ANSWERS AND MARK SCHEME

1. (a) A mathematical description which allocates a probability p to each value of a
   variable X reflecting the relative likelihood of that value occurring   B2
   (b) A list of all the members of a population   B1
   (c) The individual members of population, available for sampling   B1   4

2. (a) The values of X which will cause the null hypothesis to be rejected   B2
   (b) If H_0 is correct, then $P(A) = 4.2\%$. This is less than 5%, so if it   B1
   does happen, H_0 should be rejected   B1   4

3. (a) n repeated experiments, each with the same two possible outcomes
   with constant probabilities of success or fail   B2
   (b) If cards not replaced, probabilities change after each draw   B2
   (c) Replace card each time, to get $B(10, 0.25)$ $P(X = 0) = 0.0563$ B1 M1 A1
   (d) $P(X \geq 4) = 1 - 0.7759 = 0.224$ M1 A1
   (e) $500 \times 0.25 = 125$ M1 A1   11

4. (a) Mean $= 226/100 = 2.26$ Variance $= 734/100 - 2.26^2 = 2.23$ M1 A1 M1 A1
   Since mean $\approx$ variance, distribution could be Poisson   B1
   (b) In $Po(2.26)$, $P(X > 6) = 0.00623 + 0.00176 + 0.00044... = 0.009$ M1 M1 A1 A1
   (c) In a sample of 100, expected no. $= 100 \times 0.009 = 0.9$ intervals   M1 A1   11

5. (a) $k(2 - a)^2 = 0$, so $a = 2$ $k(6 - a)^2 = 1$, so $k = \frac{1}{16}$ M1 A1 M1 A1
   (b) Need $\frac{1}{16}(x - 2)^2 = \frac{1}{2}$ $x - 2 = \sqrt{8} = 2\sqrt{2}$ $x = 2 + 2\sqrt{2}$ M1 M1 A1 A1
   (c) $P(X > 4) = 1 - F(4) = 1 - \frac{1}{4} = \frac{3}{4}$ M1 A1
       $P(X > 5) = 1 - F(5) = 1 - \frac{9}{16} = \frac{7}{16}$ $P(X > 5 \mid X > 4) = \frac{7}{12}$ M1 A1 M1 A1   14

6. (a) If Oldies correct, take $H_0 : P(\text{Trendie}) = 0.45$. Assuming this,   B1
   distribution of Trendies is $X \sim (20, 0.45)$; then $P(X \geq 11) =$ M1 A1
   $1 - 0.7507 = 0.249 > 5\%$, so do not reject H_0 M1 A1 A1
   (b) No. of Trendies is $B(200, 0.55) \approx N(110, 49.5)$ B1 M1 A1
   so $P(X < 100) = P(X < 99.5) = P(Z < -10.5/\sqrt{49.5})$ M1 A1 M1 A1
   $= P(Z < -1.49) = 0.0681$ A1   14

7. (a) $\int_0^{10} f(x)\, dx = \frac{3}{7} + \frac{6}{875} \int_5^{10} 10x - x^2\, dx = \frac{3}{7} + \frac{6}{875} \left[5x^2 - \frac{1}{3}x^3\right]_5^{10}$ M1 A1 A1
   $= \frac{3}{7} + \frac{6}{875} \left[500 - \frac{1000}{3} - 125 + \frac{125}{3}\right] = \frac{3}{7} + \frac{4}{7} = 1$ M1 A1
   Also $f(x) \geq 0$ for all x, so f is a p.d.f.   A1
   (b) $P(X < 1) = \frac{1}{2} \times 1 \times \frac{6}{175} = \frac{3}{175}$ M1 A1
   (c) $F(x) = 0$ $(x < 0)$ $F(x) = \frac{3}{175}x^2$ $(0 \leq x \leq 5)$ B1 M1 A1
   $F(x) = \frac{3}{7} + \int_5^x \frac{6}{875}(10u - u^2)\, du = \frac{30x^2 - 2x^3 - 125}{875}$ $(5 < x \leq 10)$ M1 A1 A1
   $F(x) = 1$ $(x > 10)$ B1
   (d) $F(7) - F(2) = \frac{659}{875} - \frac{12}{175} = \frac{599}{875}$ M1 A1   17