1. (a) Briefly explain what is meant by a sample space.   **(2 marks)**

    (b) State two properties which a function f(x) must have to be a probability function.

    (2 marks)

2. A company makes two cars, model A and model B. The distance that model A travels on 10 litres of petrol is normally distributed with mean 109 km and variance 72·25 km². The distance that model B travels on 10 litres of petrol is normally distributed with mean 108·5 km and variance 169 km².

    In a trial, one of each model is filled with 10 litres of petrol and sent on a journey of 110 km. Find which model has the greater probability of completing this journey, and state the value of this probability.   **(8 marks)**

3. A, B and C are three events such that P(A) = x, P(B) = y and P(C) = $x + y$.

    It is known that P($A \cup B$) = 0·6 and P($B | A$) = 0·2.

    (a) Show that $4x + 5y = 3$.   **(2 marks)**

    It is also known that B and C are mutually exclusive and that P($B \cup C$) = 0·9

    (b) Obtain another equation in x and y and hence find the values of x and y.   **(4 marks)**

    (c) Deduce whether or not A and B are independent events.   **(2 marks)**

4. The discrete random variable X has the following probability distribution :

x	0	1	2	3	4	5
P($X = x$)	0·11	0·17	0·2	0·13	p	p^2

    (a) Find the value of p.   **(4 marks)**

    (b) Find (i) P($0 < X \le 2$),   (ii) P($X \ge 3$).   **(3 marks)**

    (c) Find the mean and the variance of X.   **(3 marks)**

    (d) Construct a table to represent the cumulative distribution function F(x).   **(2 marks)**

5. The following marks out of 50 were given by two judges to the contestants in a talent contest:

Contestant	A	B	C	D	E	F	G	H
Judge 1 (x)	43	32	40	21	47	11	29	38
Judge 2 (y)	39	25	40	22	36	13	27	32

    Given that $\sum x = 261$, $\sum x^2 = 9529$ and $\sum xy = 8373$,

    (a) calculate the product-moment correlation coefficient between the two judges' marks.

    (5 marks)

5 continued ...

    (b) Find an equation of the regression line of x on y.   **(4 marks)**

Contestant I was awarded 45 marks by Judge 2.

    (c) Estimate the mark that this contestant would have received from Judge 1.   **(2 marks)**

    (d) Comment, with explanation, on the probable accuracy of your answer.   **(2 marks)**

6. 1000 houses were sold in a small town in a one-year period. The selling prices were as given in the following table:

Selling Price	Number of Houses	Selling Price	Number of Houses
Up to £50 000	60	Up to £150 000	642
Up to £75 000	227	Up to £200 000	805
Up to £100 000	305	Up to £500 000	849
Up to £125 000	414	Up to £750 000	1000

    (a) Name (do not draw) a suitable type of graph for illustrating this data.   **(1 mark)**

    (b) Use interpolation to find estimates of the median and the quartiles.   **(6 marks)**

    (c) Estimate the 37th percentile.   **(2 marks)**

Given further that the lowest price was £42 000 and the range of the prices was £690 000,

    (d) draw a box plot to represent the data. Show your scale clearly.   **(4 marks)**

In another town the median price was £149 000, and the interquartile range was £90 000.

    (e) Briefly compare the prices in the two towns using this information.   **(2 marks)**

7. The random variable X, which can take any value from $\{1, 2, ..., n\}$, is modelled by the discrete uniform distribution with mean 10.

    (a) Show that $n = 19$ and find the variance of X.   **(4 marks)**

    (b) Find $P(3 < X \le 6)$.   **(2 marks)**

The random variable Y is defined by $Y = 3(X - 10)$.

    (c) State the mean and the variance of Y.   **(3 marks)**

The model for the distribution of X is found to be unsatisfactory, and in a refined model the probability distribution of X is taken to be

$$f(x) = \begin{cases} k(x+1) & x = 1, 2, ..., 19, \\ 0 & \text{otherwise.} \end{cases}$$

    (d) Show that $k = \dfrac{1}{209}$.   **(3 marks)**

    (e) Find $P(3 < X \le 6)$ using this model.   **(3 marks)**