GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics

Module P6

Paper D

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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P6 Paper D – Marking Guide

1. assume true for $n = k$ $\therefore \dfrac{d^k y}{dx^k} = \dfrac{k!}{(1-x)^{k+1}}$

$\therefore \dfrac{d^{k+1} y}{dx^{k+1}} = -(k+1)k!(-1)(1-x)^{-(k+2)}$ M1 A1

$= \dfrac{k!(k+1)}{(1-x)^{k+2}} = \dfrac{(k+1)!}{(1-x)^{[k+1]+1}}$ M1 A1

$\therefore$ true for $n = k + 1$ if true for $n = k$

if $n = 1$, $\dfrac{d^1 y}{dx^1} = \dfrac{1!}{(1-x)^{1+1}} = \dfrac{1}{(1-x)^2}$ M1

$y = \dfrac{1}{1-x}$, $\dfrac{dy}{dx} = -(-1)(1-x)^{-2} = \dfrac{1}{(1-x)^2}$ $\therefore$ true for $n = 1$ A1

$\therefore$ by induction true for $n \in \mathbb{Z}^+$ A1 **(7)**

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2. *(a)* $\dfrac{y_1 - y_{-1}}{2h} = x_0^2 + y_0 + 2$ M1

$y_1 = 2hx_0^2 + 2hy_0 + 4h + y_{-1}$ or $y_2 = 2hx_1^2 + 2hy_1 + 4h + y_0$ A1

$x_0 = 0, x_1 = h, x_2 = 2h; y_0 = 0, y_1 = 2h, y_2 = ?$

$y_2 = 2h(h^2) + 2h(2h) + 4h + 0 = 2h^3 + 4h^2 + 4h$ M1 A1

(b) $y_3 = 2hx_2^2 + 2hy_2 + 4h + y_1$ B1

$x_1 = h, x_2 = 2h, x_3 = 3h;\ y_1 = 2h, y_2 = 2h^3 + 4h^2 + 4h, y_3 = ?$

$y_3 = 2h(2h)^2 + 2h(2h^3 + 4h^2 + 4h) + 4h + 2h$ M1

$= 8h^3 + 4h^4 + 8h^3 + 8h^2 + 6h = 2h(2h^3 + 8h^2 + 4h + 3)$ A1

(c) $h = 0.1, y_3 = 0.2(0.002 + 0.08 + 0.4 + 3) = 0.6964$ M1 A1 **(9)**

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3. *(a)* using quad. formula $z^3 = \dfrac{\sqrt{3} \pm \sqrt{3-4}}{2}$ M1

$\therefore z^3 = \frac{\sqrt{3}}{2} \pm \frac{1}{2}\mathrm{i}$ M1 A1

(b) if $z^3 = \frac{\sqrt{3}}{2} + \frac{1}{2}\mathrm{i}$, $(re^{\mathrm{i}\theta})^3 = 1e^{\mathrm{i}\frac{\pi}{6}}$ M1 A1

$r^3 = 1$ so $r = 1$

$3\theta = 2n\pi + \frac{\pi}{6}$ M1

$n = -1, 0, 1$ gives $\theta = -\frac{11\pi}{18}, \frac{\pi}{18}, \frac{13\pi}{18}$ A1

if $z^3 = \frac{\sqrt{3}}{2} - \frac{1}{2}\mathrm{i}$, $(re^{\mathrm{i}\theta})^3 = 1e^{-\mathrm{i}\frac{\pi}{6}}$ M1

$r = 1, 3\theta = 2n\pi - \frac{\pi}{6}$

$n = -1, 0, 1$ gives $\theta = -\frac{13\pi}{18}, -\frac{\pi}{18}, \frac{11\pi}{18}$ A1

$\therefore z = e^{\pm\mathrm{i}\frac{\pi}{18}}, e^{\pm\mathrm{i}\frac{11\pi}{18}}, e^{\pm\mathrm{i}\frac{13\pi}{18}}$ A1 **(10)**

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4. *(a)* $e^{x^2} = 1 + x^2 + \frac{1}{2}x^4 + \ldots$ M1 A1

(b) $(1 + 2x)^{-1}$

$= 1 + (-1)(2x) + \frac{(-1)(-2)}{2}(2x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(2x)^3 + \frac{(-1)(-2)(-3)(-4)}{4\times 3\times 2}(2x)^4 + \ldots$ M1

$= 1 - 2x + 4x^2 - 8x^3 + 16x^4 + \ldots$ A1

$\frac{e^{x^2}}{1+2x} = (1 + x^2 + \frac{1}{2}x^4 + \ldots)(1 - 2x + 4x^2 - 8x^3 + 16x^4 + \ldots)$ M1

$= 1 - 2x + 4x^2 - 8x^3 + 16x^4 + x^2 - 2x^3 + 4x^4 + \frac{1}{2}x^4 + \ldots$ M1

$= 1 - 2x + 5x^2 - 10x^3 + \frac{41}{2}x^4 + \ldots$ A1

(c) area $\approx \int_0^{0.2} 1 - 2x + 5x^2 - 10x^3 + \frac{41}{2}x^4 \; dx$ M1

$= [\, x - x^2 + \frac{5}{3}x^3 - \frac{5}{2}x^4 + \frac{41}{10}x^5 \,]_0^{0.2}$ A1

$= \frac{1}{5} - \frac{1}{25} + \frac{1}{75} - \frac{1}{250} + \frac{41}{31250} = 0.171$ (3sf) M1 A1 **(11)**

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5. *(a)* det **A** $= 2(2 + 1) - a(1 + 3) + 1(1 - 6) = 6 - 4a - 5 = 1 - 4a$ M1 A1

A is non-singular for $a \neq \frac{1}{4}$ A1

matrix of cofactors: $\begin{pmatrix} 3 & -4 & -5 \\ 1-a & -1 & 3a-2 \\ -a-2 & 3 & 4-a \end{pmatrix}$ M1 A1

$\therefore \mathbf{A}^{-1} = \frac{1}{1-4a}\begin{pmatrix} 3 & 1-a & -a-2 \\ -4 & -1 & 3 \\ -5 & 3a-2 & 4-a \end{pmatrix}$ M1 A1

(b) $a = -1,\; \mathbf{A}^{-1} = \frac{1}{5}\begin{pmatrix} 3 & 2 & -1 \\ -4 & -1 & 3 \\ -5 & -5 & 5 \end{pmatrix}$ B1

$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \mathbf{A}^{-1}\begin{pmatrix} 5 \\ -4 \\ 2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 3 & 2 & -1 \\ -4 & -1 & 3 \\ -5 & -5 & 5 \end{pmatrix}\begin{pmatrix} 5 \\ -4 \\ 2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 5 \\ -10 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ M1 A1

$\therefore$ position vector of P is $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ A1 **(11)**

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6. *(a)*	$\Pi_1 : \mathbf{r}.(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 5, \;\; \Pi_2 : \mathbf{r}.(\mathbf{i} + 4\mathbf{j} + \mathbf{k}) = -2$		
	$\therefore (2\mathbf{i} - \mathbf{j} + 3\mathbf{k}).(\mathbf{i} + 4\mathbf{j} + \mathbf{k}) = \sqrt{14}\sqrt{18}\cos\theta$	M1	
	$2 - 4 + 3 = 1 = \sqrt{14}\sqrt{18}\cos\theta$	M1 A1	
	$\therefore \cos\theta = \frac{1}{\sqrt{14}\sqrt{18}}$ giving $\theta = 86°$ (nearest degree)	A1	
(b)	$\Pi_1 : \mathbf{r}.\frac{2\mathbf{i}-\mathbf{j}+3\mathbf{k}}{\sqrt{14}} = \frac{5}{\sqrt{14}}$	B1	
	plane parallel to Π_1 through A:		
	$\mathbf{r}.(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}).(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 4 - 1 - 6 = -3$	M1	
	$\therefore \mathbf{r}.\frac{2\mathbf{i}-\mathbf{j}+3\mathbf{k}}{\sqrt{14}} = \frac{-3}{\sqrt{14}}$	A1	
	$\therefore$ distance A to $\Pi_1 = \frac{8}{\sqrt{14}}$ or $\frac{4}{7}\sqrt{14}$	A1	
(c)	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & 3 \\ 1 & 4 & 1 \end{vmatrix}$	M1	
	$= \mathbf{i}(-1 - 12) - \mathbf{j}(2 - 3) + \mathbf{k}(8 + 1) = -13\mathbf{i} + \mathbf{j} + 9\mathbf{k}$	A1	
	$\Pi_3 : \mathbf{r}.(-13\mathbf{i} + \mathbf{j} + 9\mathbf{k}) = (4\mathbf{j} - \mathbf{k}).(-13\mathbf{i} + \mathbf{j} + 9\mathbf{k}) = 4 - 9 = -5$	M1	
	$\therefore \Pi_3 : \mathbf{r}.(-13\mathbf{i} + \mathbf{j} + 9\mathbf{k}) = -5$	A1	
	giving $-13x + y + 9z = -5$ or $13x - y - 9z = 5$	A1	**(13)**

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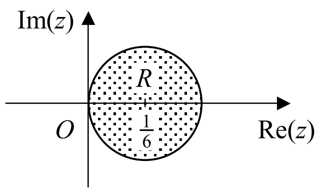
7. *(a)*	$\text{Re}(z) = 5 \;\; \therefore u + \text{i}v = \frac{1}{5 - \text{i}y - 2} = \frac{1}{3 - \text{i}y}$	M1	
	$(u + \text{i}v)(3 - \text{i}y) = 1$	A1	
	$3u + vy + \text{i}(3v - uy) = 1$		
	$\therefore 3u + vy = 1; \;\; 3v - uy = 0$	M1	
	giving $y = \frac{1-3u}{v} = \frac{3v}{u}$	A1	
	$\therefore \;\; u - 3u^2 = 3v^2; \;\; u^2 + v^2 - \frac{1}{3}u = 0$	M1	
	$(u - \frac{1}{6})^2 + v^2 = \frac{1}{36}$	A1	
	$\therefore$ circle, centre $\frac{1}{6} + 0\text{i}$, radius $\frac{1}{6}$	A1	
(b)	e.g. if $z = 6$, $z^* = 6$, $w = \frac{1}{4}$ which is inside circle	B1	
		B2	
(c)	$\arg(z - 2) = \frac{\pi}{4} \;\; \therefore \arg(z^* - 2) = -\frac{\pi}{4}$	M1 A1	
	$\therefore \arg w = \arg 1 - \arg(z^* - 2) = 0 - (-\frac{\pi}{4}) = \frac{\pi}{4}$	M1	
	image is half-line $\arg w = \frac{\pi}{4}$	A1	**(14)**

Total **(75)**

Performance Record – P6 Paper D

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	proof by induction	step-by-step soln. of diff. eqn.	complex nos.	Maclaurin series	matrices, inverse	vectors	complex trans.	
Marks	7	9	10	11	11	13	14	75
Student								