1. Find the value of λ for which $\begin{pmatrix} 2+\lambda & 1+\lambda & 3+2\lambda \\ 3+\lambda & 2+\lambda & 5+2\lambda \\ 3+\lambda & 2+\lambda & 6+3\lambda \end{pmatrix}$ is a singular matrix. **(4 marks)**

2. (a) With the usual notation, derive the approximation formula $\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$. **(3 marks)**

   (b) Given that $\frac{dy}{dx} = 2x - y$ and that $y = 1$ when $x = 0$, use the above formula with $h = 0{\cdot}1$ to estimate the value of y when $x = 0{\cdot}2$. **(5 marks)**

3. A complex number z satisfies the equation $|z - i| = 2|z + 1|$.

   (a) Show that this equation represents a circle in the Argand diagram and find the centre and radius of this circle. **(8 marks)**

   (b) Write in the form $w = f(z)$ a transformation which would map this circle onto another circle with the same radius but with its centre at the origin. **(2 marks)**

4. (a) Prove by induction that for any positive integer n, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$. **(6 marks)**

   (b) Given that $(\cos\theta + i\sin\theta)^4 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$, find all the possible values of θ between 0 and 2π. **(5 marks)**

5. (a) Find the first three derivatives of e^{3x}. **(2 marks)**

   Hence conjecture a formula for the nth derivative of e^{3x}, and use mathematical induction to prove your conjecture. **(5 marks)**

   (b) Write down, as far as the terms in x^3, the Maclaurin series for e^{3x} and for $\ln(1 + 2x)$. **(2 marks)**

   (c) By multiplying these series together, or otherwise, find a series expansion for $e^{3x}\ln(1 + 2x)$ as far as the term in x^3. **(4 marks)**

6. The points A, B, C and D have position vectors $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $-4\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$, $2\mathbf{j} + 2\mathbf{k}$ and $6\mathbf{i} + \mathbf{j} + 6\mathbf{k}$ respectively.

   (a) Show that $ABCD$ is a parallelogram. **(2 marks)**

   (b) Find $\overrightarrow{AB} \times \overrightarrow{AD}$. **(3 marks)**

   (c) Find a vector equation of the plane containing A, B, C and D. **(4 marks)**

   (d) Find the perpendicular distance from the origin to the plane $ABCD$. **(2 marks)**

   (e) Calculate the volume of the pyramid $OABCD$. **(3 marks)**

7. A linear transformation T is represented by the matrix

$$\mathbf{M} = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ -1 & 0 & 2 \end{pmatrix}.$$

   The points P and Q are mapped to P' and Q' respectively by T.

   (a) Given that P is the point $(1, 3, 2)$, find the co-ordinates of P'. **(1 mark)**

   (b) Given that Q' is the point $(5, -3, 0)$, find the co-ordinates of Q. **(3 marks)**

   (c) Show that 1 is the only real eigenvalue of T and find an eigenvector of magnitude 1. **(7 marks)**

   Another transformation S has matrix $\mathbf{M}^{\mathrm{T}}$.

   (d) Find the image of P under the combined transformation ST. **(4 marks)**