1. Find the first three terms of the Maclaurin series for $\tan\left(x + \frac{\pi}{4}\right)$. **(5 marks)**

2. Find the four fourth roots of 16i, in the form $re^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. **(6 marks)**

3. O is the origin and A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively relative to O, where $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + 3\mathbf{j} - \mathbf{k}$.
   (a) Find $\mathbf{a} \times \mathbf{b}$. **(2 marks)**
   (b) Hence write down the area of the triangle OAB. **(2 marks)**
   (c) Find, in the form $(\mathbf{r} - \mathbf{u}) \times \mathbf{v} = \mathbf{0}$, an equation of the straight line through A and B. **(3 marks)**

4. Given that $\mathbf{M} = \begin{pmatrix} x & x \\ x & x \end{pmatrix}$, use the method of induction to prove that for any integer $n \geq 1$,

$$\mathbf{M}^n = \begin{pmatrix} 2^{n-1}x^n & 2^{n-1}x^n \\ 2^{n-1}x^n & 2^{n-1}x^n \end{pmatrix}.$$

(8 marks)

5. Given that $\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 3y = 4$, and that $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$,
   use the Taylor's series method to express y as a polynomial in x, as far as the term in x^3.
   Hence find the approximate value of y when $x = 0{\cdot}15$, correct to 3 decimal places. **(9 marks)**

6. The Cartesian equation of a plane is $6x - 5y + 4z = 17$.
   (a) Find the equation of this plane in the vector form $\mathbf{r} \cdot \mathbf{n} = p$. **(2 marks)**
   (b) Find the perpendicular distance from the origin to this plane. **(3 marks)**
   (c) Find, to the nearest degree, the angle between this plane and the plane $\mathbf{r} \cdot (3\mathbf{i} - \mathbf{j} - \mathbf{k}) = 2$. **(5 marks)**

7. Given that the transformation T is represented by the matrix

$$\mathbf{M} = \begin{pmatrix} 5 & 0 & 1 \\ 6 & -1 & 0 \\ 2 & -8 & -2 \end{pmatrix},$$

find

(a) the inverse of **M**, **(5 marks)**

(b) the coordinates of the point which is mapped to $(2, 4, -8)$ by T. **(3 marks)**

Given that -4 is an eigenvalue of **M**,

(c) find the other eigenvalue of **M**. **(6 marks)**

8. (a) Use de Moivre's theorem to show that if $z = \cos\theta + i\sin\theta$ and n is a positive integer then

$$z^n + \frac{1}{z^n} = 2\cos n\theta \quad \text{and} \quad z^n - \frac{1}{z^n} = 2i\sin n\theta.$$ **(5 marks)**

(b) Hence express $\cos^4\theta$ in terms of cosines of multiples of θ. **(7 marks)**

(c) By using a standard formula for $\cos 2\theta$ in your result, deduce an expression for $\cos 4\theta$ in terms of powers of $\cos\theta$. **(4 marks)**