PURE MATHS 6 (A) TEST PAPER 6 : ANSWERS AND MARK SCHEME

1. $u_1 = 1 + 0 = 2 - 1$: true for $n = 1$. Assume true for $n = k$: $u_k = 2^k - 1$ B1 M1
   Then $u_{k+1} = 1 + 2(2^k - 1) = 2^{k+1} - 1$, so true for $n = k + 1$, etc. A1 A1 — 4

2. Char. eqn. is $(2 - \lambda)(-1 - \lambda) - 10 = 0$ $\lambda^2 - \lambda - 12 = 0$ M1 A1
   $(\lambda + 3)(\lambda - 4) = 0$ Eigenvalues are -3 and 4 M1 A1 A1 — 5

3. $f(0) = 1$ $f'(x) = e^x(\cos \pi x - \pi \sin \pi x)$ $f'(0) = 1$ M1 A1
   $f''(x) = e^x(\cos \pi x - \pi \sin \pi x + \pi \sin \pi x - \pi^2 \cos \pi x)$ $f''(0) = 1 - \pi^2$ M1 A1
   $f(x) = 1 + x + \frac{1}{2}(1 - \pi^2)x^2 + \ldots$ A1
   $x = 1/10$: $f(x) \approx 1{\cdot}1 + (1 - \pi^2)/200 = 1{\cdot}06$ M1 A1 — 7

4. (a) $z^4 = 32i$ $z^2 = 4\sqrt{2}\sqrt{i} = 4\sqrt{2}\,[\pm(1 + i)/\sqrt{2})] = \pm(4 + 4i)$ M1 A1 A1
   (b) $z = 2(1 + i)^{1/2}$ or $2(-1 - i)^{1/2} = 2\,(\sqrt{2})^{1/2}\,(\cos \pi/4 + i \sin \pi/4)^{1/2}$ etc. M1 M1
   $z = 2^{5/4}\,(\cos \pi/8 + i \sin \pi/8)$, $2^{5/4}\,(\cos 5\pi/8 + i \sin 5\pi/8)$, A1 A1
   $2^{5/4}\,(\cos -7\pi/8 + i \sin -7\pi/8)$, $2^{5/4}\,(\cos -3\pi/8 + i \sin -3\pi/8)$ A1 A1 — 9

5. (a) $(1, 0, 0) \to (-2, 1, 3)$, $(0, 1, 0) \to (1, 0, -4)$, $(0, 0, 1) \to (1, -1, 2)$ M1 A1
   Matrix is $\mathbf{M} = \begin{pmatrix} -2 & 1 & 1 \\ 1 & 0 & -1 \\ 3 & -4 & 2 \end{pmatrix}$ A1
   (b) Det $\mathbf{M} = 8 - 5 - 4 = -1$ M1 A1
   $\mathbf{M}^{-1} = \frac{1}{-1}\begin{pmatrix} -4 & -6 & -1 \\ -5 & -7 & -1 \\ -4 & -5 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 6 & 1 \\ 5 & 7 & 1 \\ 4 & 5 & 1 \end{pmatrix}$ M1 A1 A1
   (c) Applying $\mathbf{M}^{-1}$ to $(2, -1, 0)$ gives $(2, 3, 3)$ M1 A1 — 10

6. (a) If $w = z$, $z^2 + 1 = z$ $z^2 - z + 1 = 0$ $z = 1/2 \pm i\sqrt{3}/2$ M1 A1 M1 A1
   Points are $e^{i\pi/3}$, $e^{-i\pi/3}$ A1 A1
   (b) $x = 1$ $u + iv = (1 + iy)^2 + 1 = 2 - y^2 + 2iy$ M1 M1 A1
   $u = 2 - y^2$, $v = 2y$ $v^2 = 8 - 4u$, which is a parabola M1 A1 A1 — 12

7. (a) $y'_0 = 2 - 4 = -2$ $(y_1 - y_{-1})/0{\cdot}2 = -2$ M1 A1 M1
   $y_1 - y_{-1} = -0{\cdot}4$ $y_1 = y_{-1} - 0{\cdot}4$ A1
   (b) $-2 = (-1 - y_{-1})/0{\cdot}1$ $y_{-1} = -1 + 0{\cdot}2 = -0{\cdot}8 = y(0{\cdot}9)$ M1 A1 A1
   (c) $y(1{\cdot}1) = y_1 = -1{\cdot}2$ $y'_1 = 2(1{\cdot}1) + 4(1{\cdot}1)(-1{\cdot}2) = -3{\cdot}08$ A1 M1 A1
   $-3{\cdot}08 \approx (y_2 + 1)/0{\cdot}2$ $y(1{\cdot}2) = -0{\cdot}616 - 1 = -1{\cdot}616$ M1 M1 A1 — 13

8. (a) $x = 1 + 3\lambda - \mu$, $y = 2 - \lambda + \mu$, $z = \lambda - 1$ B2
   $x + y = 3 + 2\lambda = 3 + 2(z + 1)$ $x + y - 2z = 5$ M1 A1
   $\mathbf{r} \cdot (\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 5$ Unit vector normal $= (\mathbf{i} + \mathbf{j} - 2\mathbf{k})/\sqrt{6}$ A1 M1 A1
   (b) Distance $= 5/\sqrt{6}$ A1
   (c) $32 + 3 - 10 = 20 + 30 - 25 = 25$, $8 + 1 - 4 = 5 + 10 - 10 = 5$ B1 B1
   (d) Line is $\mathbf{r} = 8\mathbf{i} + \mathbf{j} + 2\mathbf{k} + t(\mathbf{i} - 3\mathbf{j} - \mathbf{k})$, or equivalent M1 A1
   (e) Angle between normals: $\cos\theta = 17/\sqrt{300} = 0{\cdot}9815$ $\theta = 11°$ M1 A1 A1 — 15