

# GCE Examinations

# Pure Mathematics

# Module P5

Advanced Subsidiary / Advanced Level

## Paper G

Time: 1 hour 30 minutes

### *Instructions and Information*

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Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

### *Advice to Candidates*

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You must show sufficient working to make your methods clear to an examiner.  
Answers without working will gain no credit.



*Written by Rosemary Smith & Shaun Armstrong*

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1. Given that  $y = e^{\arctan x}$ ,

(a) find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ . **(4 marks)**

The curve  $y = e^{\arctan x}$  has a point of inflexion.

(b) Find the coordinates of this point of inflexion. **(3 marks)**

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2. (a) Prove that

$$\frac{d}{dx}(\operatorname{arcosh} x) = \frac{1}{\sqrt{x^2 - 1}}. \quad \text{(3 marks)}$$

(b) Find

$$\int \operatorname{arcosh} x \, dx. \quad \text{(4 marks)}$$

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3. Find

$$\int_0^{\frac{\pi}{4}} \frac{1}{1 + \sin 2x} \, dx. \quad \text{(8 marks)}$$

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4. (a) Find

$$\int \frac{1}{\sqrt{4x^2 - 4x + 10}} dx. \quad (6 \text{ marks})$$

- (b) Hence evaluate

$$\int_{\frac{1}{2}}^2 \frac{1}{\sqrt{4x^2 - 4x + 10}} dx,$$

giving your answer in terms of natural logarithms. (3 marks)

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5. (a) On the same axes sketch the curves with equations  $y = 2 - \tanh x$  and  $y = 3 \operatorname{sech} x$ , giving the coordinates of the points of intersection of the curves with the coordinate axes and the equations of the asymptotes. (5 marks)

- (b) Solve the equation

$$2 - \tanh x = 3 \operatorname{sech} x,$$

giving your answers to 2 decimal places. (7 marks)

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6. 
$$I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, dx, \quad n \geq 0.$$

- (a) Show that

$$I_n = \frac{n-1}{n} I_{n-2}, \quad n \geq 2. \quad (7 \text{ marks})$$

The curve  $C$  is defined by  $y = \sin^2 x$ ,  $0 \leq x \leq \pi$ .

The area bounded by  $C$  and the positive  $x$ -axis is rotated through  $2\pi$  radians about the  $x$ -axis.

- (b) Find the volume of the solid generated giving your answer in terms of  $\pi$ . (7 marks)
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*Turn over*

7.

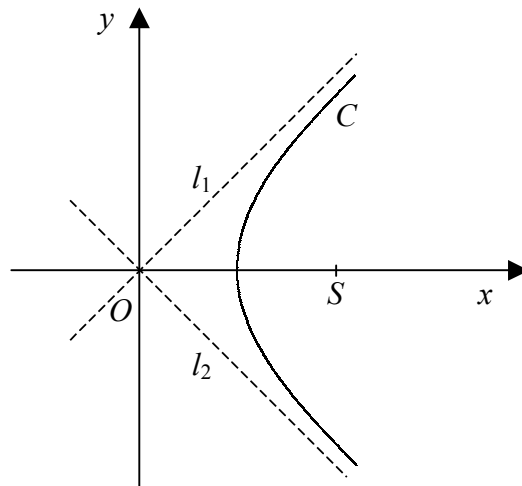


Fig. 1

Figure 1 shows the curve  $C$  which is part of the hyperbola with parametric equations

$$x = a \cosh t, \quad y = 2a \sinh t,$$

where  $a$  is a positive constant and  $x \geq a$ . The lines  $l_1$  and  $l_2$  are asymptotes to  $C$ .

(a) Show that the radius of curvature of  $C$  at its vertex is  $4a$ . (6 marks)

(b) Show that an equation of the tangent to  $C$  at the point  $P(a \cosh p, 2a \sinh p)$  is

$$2x \cosh p - y \sinh p = 2a. \quad \text{(4 marks)}$$

The tangent to the curve  $C$  at  $P$  meets the asymptote  $l_1$  at  $Q$ .

Given that  $QS$  is parallel to the  $y$ -axis, where  $S$  is the focus,

(c) show that  $p = \frac{1}{2} \ln 5$ . (8 marks)

**END**