

GCE Examinations
Advanced Subsidiary / Advanced Level
Pure Mathematics
Module P5

Paper F

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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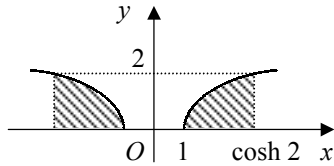
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P5 Paper F – Marking Guide

1. let $y = \operatorname{artanh}(\sin x) \therefore \tanh y = \sin x$
 $\therefore \operatorname{sech}^2 y \frac{dy}{dx} = \cos x$ M1
 $\operatorname{sech}^2 y = 1 - \tanh^2 y = 1 - \sin^2 x = \cos^2 x$ M1 A1
 $\therefore \frac{dy}{dx} = \frac{\cos x}{\cos^2 x} = \sec x$ A1 (4)
-
2. $y = \ln(\sec x) \therefore \frac{dy}{dx} = \frac{1}{\sec x} \times \sec x \tan x = \tan x$ M1 A1
 $s = \int_0^{\frac{\pi}{3}} \sqrt{1 + \tan^2 x} \, dx$ M1
 $= \int_0^{\frac{\pi}{3}} \sec x \, dx$ A1
 $= [\ln |\sec x + \tan x|]_0^{\frac{\pi}{3}}$ A1
 $= \ln(2 + \sqrt{3}) - \ln(1 + 0) = \ln(2 + \sqrt{3})$ M1 A1 (7)
-
3. $x = t^2, \frac{dx}{dt} = 2t, \frac{d^2x}{dt^2} = 2; y = t^3, \frac{dy}{dt} = 3t^2, \frac{d^2y}{dt^2} = 6t$ M1 A2
 at $(1, 1) \ t = 1 \therefore \frac{dx}{dt} = 2, \frac{d^2x}{dt^2} = 2, \frac{dy}{dt} = 3, \frac{d^2y}{dt^2} = 6$ M1 A1
 $\rho = \frac{(2^2 + 3^2)^{\frac{3}{2}}}{2 \times 6 - 2 \times 3} = \frac{13\sqrt{13}}{6}$ M1 A1 (7)
-
4. (a) $u = (\ln x)^n, u' = n(\ln x)^{n-1} \times \frac{1}{x}; v' = 1, v = x$ M1
 $I_n = [x(\ln x)^n]_1^e - \int_1^e n(\ln x)^{n-1} \, dx$ A1
 $I_n = e - 0 - nI_{n-1} = e - nI_{n-1}$ M1 A1
- (b) $I_0 = \int_1^e dx = [x]_1^e = e - 1$ M1 A1
 $I_1 = e - I_0 = e - (e - 1) = 1$ M1
 $I_2 = e - 2I_1 = e - 2$ M1
 $I_3 = e - 3I_2 = e - 3(e - 2) = 6 - 2e$ A1 (9)
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5. $x = \cosh 2, y = 2$ B1



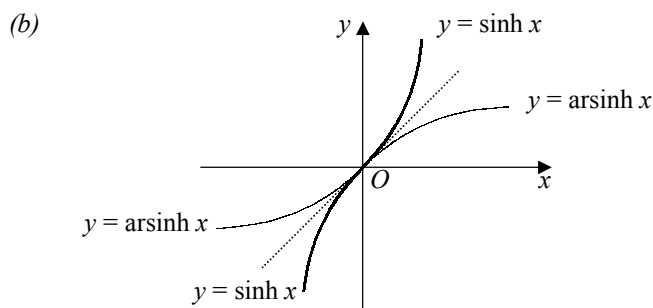
$$\begin{aligned} \text{vol. of cylinder} &= \pi(\cosh 2)^2 \times 2 = 2\pi \cosh^2 2 && \text{M1 A1} \\ \text{vol. to be removed} &= \int_0^2 \pi x^2 dy = \int_0^2 \pi \cosh^2 y dy && \text{M1} \\ &= \frac{\pi}{2} \int_0^2 (1 + \cosh 2y) dy && \text{M1} \\ &= \frac{\pi}{2} \left[y + \frac{1}{2} \sinh 2y \right]_0^2 && \text{A1} \\ &= \frac{\pi}{2} \left[2 + \frac{1}{2} \sinh 4 - (0 + 0) \right] = \pi \left(1 + \frac{1}{4} \sinh 4 \right) && \text{M1 A1} \\ \text{vol. required} &= \pi(2 \cosh^2 2 - 1 - \frac{1}{4} \sinh 4) = 20.5\pi \therefore a = 20.5 \text{ (1dp)} && \text{M1 A1 (10)} \end{aligned}$$

6. (a) $\frac{3x-7}{(x+1)(x^2+4)} \equiv \frac{A}{x+1} + \frac{Bx+C}{x^2+4}$ M1
 $3x-7 \equiv A(x^2+4) + (Bx+C)(x+1)$ M1 A2
giving $A = -2, B = 2, C = 1$
 $f(x) \equiv \frac{2x+1}{x^2+4} - \frac{2}{x+1}$

(b) $\int_0^2 f(x) dx = \int_0^2 \left(\frac{2x}{x^2+4} + \frac{1}{x^2+4} - \frac{2}{x+1} \right) dx$ M1
 $= \left[\ln |x^2+4| + \frac{1}{2} \arctan \left(\frac{x}{2} \right) - 2 \ln |x+1| \right]_0^2$ M1 A2
 $= \ln 8 + \frac{\pi}{8} - 2 \ln 3 - (\ln 4 + 0 - 0)$ M1 A1
 $= \ln \frac{8}{9 \times 4} + \frac{\pi}{8} = \frac{\pi}{8} + \ln \frac{2}{9}$ A1 (11)

7. (a) $\frac{2x}{a^2} + \frac{2y}{b^2} \frac{dy}{dx} = 0 \therefore \frac{dy}{dx} = -\frac{xb^2}{ya^2}$ M1 A1
- at P $\frac{dy}{dx} = -\frac{a \cos \theta \times b^2}{b \sin \theta \times a^2} = -\frac{b \cos \theta}{a \sin \theta}$ M1
- \therefore eqn. of normal is
- $$y - b \sin \theta = \frac{a \sin \theta}{b \cos \theta} (x - a \cos \theta)$$
- M1 A1
- or $ax \sin \theta - by \cos \theta = (a^2 - b^2) \sin \theta \cos \theta$
- (b) at Q, $y = 0$, $ax \sin \theta = (a^2 - b^2) \sin \theta \cos \theta \therefore x = \frac{(a^2 - b^2) \cos \theta}{a}$ M1 A1
- at R, $x = a \cos \theta$ B1
- $$\therefore \frac{OQ}{OR} = \frac{(a^2 - b^2) \cos \theta}{a} \times \frac{1}{a \cos \theta} = \frac{a^2 - b^2}{a^2} = 1 - \frac{b^2}{a^2}$$
- M1 A1
- $$b^2 = a^2(1 - e^2) \therefore \frac{b^2}{a^2} = 1 - e^2 \therefore \frac{OQ}{OR} = 1 - (1 - e^2) = e^2$$
- M1 A1 (12)

8. (a) let $y = \operatorname{arsinh} x \therefore x = \sinh y = \frac{1}{2}(e^y - e^{-y})$ M1
- $$\therefore e^y - e^{-y} = 2x$$
- $$e^{2y} - 2xe^y - 1 = 0$$
- A1
- $$e^y = \frac{2x \pm \sqrt{4x^2 + 4}}{2} = x \pm \sqrt{x^2 + 1}$$
- M1 A1
- $$\sqrt{x^2 + 1} > x \therefore x - \sqrt{x^2 + 1} < 0 \text{ but } e^y > 0$$
- M1
- $$\therefore e^y = x + \sqrt{x^2 + 1}$$
- $$\therefore y = \operatorname{arsinh} x = \ln(x + \sqrt{x^2 + 1})$$
- A1



- (c) $x = \sinh [\ln(3x - 2)] = \frac{1}{2} [e^{\ln(3x-2)} - e^{-\ln(3x-2)}]$ M1
- $$2x = 3x - 2 - \frac{1}{3x-2}$$
- A1
- $$2x(3x - 2) = (3x - 2)^2 - 1$$
- M1
- $$6x^2 - 4x = 9x^2 - 12x + 4 - 1$$
- $$3x^2 - 8x + 3 = 0$$
- A1
- $$x = \frac{8 \pm \sqrt{28}}{6} = \frac{4 \pm \sqrt{7}}{3}$$
- M1
- $$x > \frac{2}{3} \therefore x = \frac{4 + \sqrt{7}}{3} \quad [= 2.22 \text{ (3sf)}]$$
- A1 (15)

Total (75)

