GCE Examinations

Pure Mathematics Module P5

Advanced Subsidiary / Advanced Level

Paper D

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Rosemary Smith & Shaun Armstrong

1. $$y = \frac{\operatorname{cosech} x}{x^2 + 1}.$$

(a) Find $\frac{dy}{dx}$. **(4 marks)**

(b) Find the value of $\frac{dy}{dx}$ when $x = 0.5$, giving your answer to 2 decimal places. **(1 mark)**

---

2. A curve has intrinsic coordinates (s, ψ) and radius of curvature ρ.

Given that $\rho = 2(s + a)$, where a is constant, show that the intrinsic equation of the curve can be written in the form

$$s = Ae^{2\psi} - a,$$

where A is constant. **(5 marks)**

---

3. (a) Prove that

$$\sinh 3x \equiv 4\sinh^3 x + 3\sinh x.$$ **(5 marks)**

(b) Hence, or otherwise, solve the equation

$$\sinh 3x = 7\sinh^2 x,$$

giving your answers in terms of natural logarithms where appropriate. **(6 marks)**

---

4. (a) Find $$\int \frac{1}{\sqrt{9 - 4x^2}}\, dx.$$ **(3 marks)**

(b) Find $$\int \frac{1 - 2x}{\sqrt{9 - 4x^2}}\, dx.$$ **(3 marks)**

(c) Hence, or otherwise, solve the differential equation

$$\sqrt{9 - 4x^2}\,\frac{dy}{dx} = y(1 - 2x),$$

given that $y = 1$ when $x = 0$. **(6 marks)**

---

5. The curve C has equation $y^2 = 4ax$, where a is a positive constant.

(a) Show that an equation of the tangent to C at the point $P\ (ap^2, 2ap)$, $p \neq 0$, is

$$yp = x + ap^2 .$$ **(4 marks)**

The point $Q\ (aq^2, 2aq)$, is on C where $q \neq 0$ and $p \neq q$. The chord PQ passes through the focus of C.

Show that

(b) $pq = -1$, **(5 marks)**

(c) the tangent to C at P and the tangent to C at Q meet on the directrix of C. **(4 marks)**

6.
$$I_n = \int_0^{\frac{\pi}{4}} \sec^n x \ \mathrm{d}x, \qquad n \geq 0.$$

(a) Show that

$$(n-1)I_n = (\sqrt{2})^{n-2} + (n-2)I_{n-2}, \qquad n \geq 2.$$ **(7 marks)**

(b) Hence find the exact value of I_3, giving your answer in terms of natural logarithms.

(6 marks)

---

7. *(a)* Show that

$$\int \sqrt{a^2 + x^2}\ \mathrm{d}x = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\operatorname{arsinh}\left(\frac{x}{a}\right) + c .$$ **(9 marks)**

The parametric equations of the curve C are

$$x = 2t, \qquad y = t^2, \qquad 0 \leq t \leq 3.$$

(b) Show that the length of C is given by

$$2\int_0^3 \sqrt{1 + t^2}\ \mathrm{d}t .$$ **(4 marks)**

(c) Find the length of C. **(3 marks)**

END