

GCE Examinations
Advanced Subsidiary / Advanced Level
Pure Mathematics
Module P5

Paper A

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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P5 Paper A – Marking Guide

1. $y = x + 2x^2 + 5x^3$, $\frac{dy}{dx} = 1 + 4x + 15x^2$, $\frac{d^2y}{dx^2} = 4 + 30x$ M1 A1

when $x = 0$, $\frac{dy}{dx} = 1$, $\frac{d^2y}{dx^2} = 4$ A1

$\rho = \frac{(1+1^2)^{\frac{3}{2}}}{4} = \frac{2\sqrt{2}}{4} = \frac{1}{\sqrt{2}}$ M1 A1 (5)

2. $u = x$, $u' = 1$; $v' = \operatorname{sech}^2 x$, $v = \tanh x$ M1

$$\int_0^{\ln 2} x \operatorname{sech}^2 x \, dx = [x \tanh x]_0^{\ln 2} - \int_0^{\ln 2} \tanh x \, dx$$

A1

$$= [x \tanh x - \ln(\cosh x)]_0^{\ln 2}$$

M1 A1

$$= (\ln 2 \times \frac{e^{\ln 2} - e^{-\ln 2}}{e^{\ln 2} + e^{-\ln 2}} - \ln\left(\frac{e^{\ln 2} + e^{-\ln 2}}{2}\right)) - (0 - \ln 1)$$

M1 A1

$$= \ln 2 \times \frac{2 - \frac{1}{2}}{2 + \frac{1}{2}} - \ln\left(\frac{2 + \frac{1}{2}}{2}\right)$$

M1

$$= \frac{3}{5} \ln 2 - \ln \frac{5}{4}$$

A1 (8)

3. (a) let $y = \arcsin 2x \therefore \sin y = 2x$

$\therefore \cos y \frac{dy}{dx} = 2$ M1

$$\frac{dy}{dx} = \frac{2}{\sqrt{1 - \sin^2 y}} = \frac{2}{\sqrt{1 - 4x^2}}$$

M1 A1

(b) $f'(x) = 2x \times \frac{2}{\sqrt{1 - 4x^2}} + 2 \arcsin 2x + \frac{1}{2}(-8x)(1 - 4x^2)^{-\frac{1}{2}}$ M1 A1

$$= \frac{4x}{\sqrt{1 - 4x^2}} + 2 \arcsin 2x - \frac{4x}{\sqrt{1 - 4x^2}} = 2 \arcsin 2x$$

A1

$$f''(x) = \frac{4}{\sqrt{1 - 4x^2}}$$

A1

$$f''(x)[f(x) - xf'(x)] = \frac{4}{\sqrt{1 - 4x^2}} (2x \arcsin 2x + \sqrt{1 - 4x^2} - 2x \arcsin 2x)$$

M1

$$= \frac{4}{\sqrt{1 - 4x^2}} \times \sqrt{1 - 4x^2} = 4$$

A1 (9)

4. (a) at A, $y = 0 \therefore \frac{1}{3}t(3 - t^2) = 0$ M1
 $t = 0$ (at O) or $\pm\sqrt{3}$; $t > 0 \therefore a = \sqrt{3}$ A1

(b) $\frac{dx}{dt} = 2t, \frac{dy}{dt} = 1 - t^2$ M1

$$A = \int_0^{\sqrt{3}} 2\pi(t - \frac{1}{3}t^3)\sqrt{4t^2 + (1-t^2)^2} dt$$

$$= 2\pi \int_0^{\sqrt{3}} (t - \frac{1}{3}t^3)\sqrt{1+2t^2+t^4} dt$$

$$= 2\pi \int_0^{\sqrt{3}} (t - \frac{1}{3}t^3)(1+t^2) dt$$

$$= 2\pi \int_0^{\sqrt{3}} t + \frac{2}{3}t^3 - \frac{1}{3}t^5 dt$$

$$= 2\pi[\frac{1}{2}t^2 + \frac{1}{6}t^4 - \frac{1}{18}t^6]_0^{\sqrt{3}}$$

$$= 2\pi[(\frac{3}{2} + \frac{3}{2} - \frac{3}{2}) - 0] = 3\pi$$

M1 A1 (10)

5. (a) $2 \cosh^2 x - 1 = 2 \times \frac{1}{4}(e^x + e^{-x})^2 - 1$ M1
 $= \frac{1}{2}(e^{2x} + 2 + e^{-2x}) - 1$ M1
 $= \frac{1}{2}(e^{2x} + e^{-2x}) + 1 - 1 = \frac{1}{2}(e^{2x} + e^{-2x}) = \cosh 2x$ A1

(b) $2(2 \cosh^2 x - 1) = 13 \cosh x - 12$ M1
 $4 \cosh^2 x - 13 \cosh x + 10 = 0$ A1
 $(4 \cosh x - 5)(\cosh x - 2) = 0$ M1
 $\cosh x = \frac{5}{4}$ or 2 A1
 $x = \ln(\frac{5}{4} + \sqrt{\frac{25}{16} - 1})$ or $\ln(2 + \sqrt{3})$ M1 A1
 $x = \ln 2$ or $\ln(2 + \sqrt{3})$ A1 (10)

6. (a) $x^2 - 10x + 41 \equiv (x - 5)^2 - 25 + 41$ M1
 $\equiv (x - 5)^2 + 16 \therefore a = -5, b = 16$ A1

(b) $\int_5^9 \frac{x}{\sqrt{x^2 - 10x + 41}} dx = \int_5^9 \frac{x}{\sqrt{(x-5)^2 + 16}} dx$

$$u = x - 5, \frac{du}{dx} = 1$$

$$= \int_0^4 \frac{u+5}{\sqrt{u^2+16}} du$$

$$= \int_0^4 \frac{u}{\sqrt{u^2+16}} + \frac{5}{\sqrt{u^2+16}} du$$

$$= [\sqrt{u^2+16} + 5 \operatorname{arsinh}(\frac{u}{4})]_0^4$$

$$= (\sqrt{32} + 5 \operatorname{arsinh} 1) - (4 + 0)$$

$$= 4\sqrt{2} - 4 + 5 \ln(1 + \sqrt{2})$$

$$= 4(\sqrt{2} - 1) + 5 \ln(1 + \sqrt{2}) \therefore p = 4, q = 5, r = 1 + \sqrt{2}$$

A1 (10)

7.	(a)	$u = x^n, u' = nx^{n-1}; v' = \cos x, v = \sin x$	M1
		$I_n = [x^n \sin x]_0^{\frac{\pi}{2}} - n \int_0^{\frac{\pi}{2}} x^{n-1} \sin x \, dx$	A1
		$u = x^{n-1}, u' = (n-1)x^{n-2}; v' = \sin x, v = -\cos x$	M1
		$I_n = [x^n \sin x]_0^{\frac{\pi}{2}} - n \{ [-x^{n-1} \cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} (n-1)x^{n-2} \cos x \, dx \}$	A1
		$I_n = (\frac{\pi}{2})^n - 0 - n(0-0) - n(n-1)I_{n-2}$	
		$I_n = (\frac{\pi}{2})^n - n(n-1)I_{n-2}$	A1
	(b)	$I_0 = \int_0^{\frac{\pi}{2}} \cos x \, dx = [\sin x]_0^{\frac{\pi}{2}} = 1$	M1 A1
		$I_2 = (\frac{\pi}{2})^2 - 2 \times 1 \times I_0 = \frac{1}{4}\pi^2 - 2$	M1 A1
		$I_4 = (\frac{\pi}{2})^4 - 4 \times 3 \times (\frac{1}{4}\pi^2 - 2) = \frac{1}{16}\pi^4 - 3\pi^2 + 24$	M1 A1 (11)

8.	(a)	$y = \frac{c^2}{x}, \frac{dy}{dx} = -\frac{c^2}{x^2}$	M1
		at P, $\frac{dy}{dx} = -\frac{c^2}{c^2 p^2} = -\frac{1}{p^2}$	A1
		eqn. is $y - \frac{c}{p} = -\frac{1}{p^2}(x - cp)$	M1
		giving $x + yp^2 = 2cp$	A1
	(b)	X is $(cq, 0)$	B1
		$\therefore cq + 0 = 2cp$ giving $q = 2p$	M1 A1
	(c)	$P(cp, \frac{c}{p}), Q(2cp, \frac{c}{2p})$	M1
		x-coord of M = $\frac{1}{2}(cp + 2cp) = \frac{3}{2}cp$	A1
		y-coord of M = $\frac{1}{2}(\frac{c}{p} + \frac{c}{2p}) = \frac{3c}{4p}$	A1
		$x \times y = \frac{3}{2}cp \times \frac{3c}{4p} \therefore$ eqn is $xy = \frac{9c^2}{8}$	M1 A1 (12)

Total (75)

