GCE Examinations

Pure Mathematics
Module P5

Advanced Subsidiary / Advanced Level

Paper F

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Rosemary Smith & Shaun Armstrong

1. $$f(x) = \text{artanh}(\sin x).$$

Show that $f'(x) = \sec x$. **(4 marks)**

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2. Find the length of the arc of the curve with equation $y = \ln(\sec x)$ between $x = 0$ and $x = \frac{\pi}{3}$, giving your answer in terms of natural logarithms.

(7 marks)

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3. A curve has parametric equations

$$x = t^2, \quad y = t^3.$$

Show that the radius of curvature of the curve at the point (1, 1) is $\frac{13\sqrt{13}}{6}$. **(7 marks)**

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4. $$I_n = \int_1^{\mathrm{e}} (\ln x)^n \; \mathrm{d}x.$$

(a) Prove that, for $n \in \mathbb{Z}^+$,

$$I_n = \mathrm{e} - nI_{n-1}.$$ **(4 marks)**

(b) Find I_3, leaving your answer in terms of e. **(5 marks)**

5.

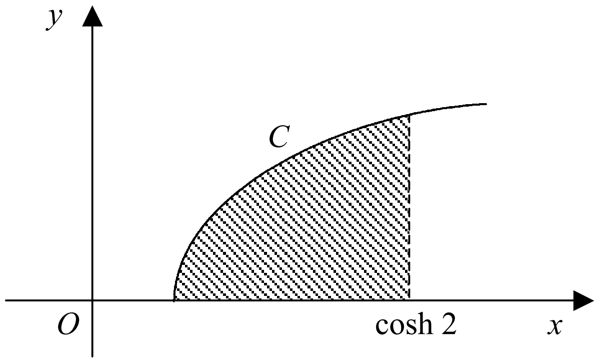

Fig. 1

Figure 1 shows the curve C which has equation $y = \text{arcosh}\, x$.

The shaded region bounded by C, the x-axis and the line $x = \cosh 2$ is rotated through 2π about the y-axis.

The volume of revolution of the solid generated is $a\pi$.

Find the value of a to one decimal place. **(10 marks)**

6.

$$\text{f}(x) \equiv \frac{3x-7}{(x+1)(x^2+4)}, \quad x \neq -1.$$

(a) Express f(x) in partial fractions. **(4 marks)**

(b) Show that

$$\int_0^2 \text{f}(x) \;\text{d}x = \frac{\pi}{8} + \ln\left(\frac{2}{9}\right).$$ **(7 marks)**

Turn over

7. The ellipse C has equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are positive constants and $a > b$.

(a) Find an equation of the normal to C at the point P $(a\cos\theta, b\sin\theta)$. **(5 marks)**

The normal to C at P meets the x-axis at Q.

R is the foot of the perpendicular from P to the x-axis.

(b) Show that $\frac{OQ}{OR} = e^2$, where e is the eccentricity of C. **(7 marks)**

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8. *(a)* Using the definitions of hyperbolic functions in terms of exponential functions prove that

$$\text{arsinh}\, x = \ln(x + \sqrt{x^2 + 1}).$$ **(6 marks)**

(b) On the same axes sketch the graphs of $y = \sinh x$ and $y = \text{arsinh}\, x$. **(3 marks)**

(c) Solve the equation

$$x = \sinh[\ln(3x - 2)], \quad x > \tfrac{2}{3}.$$ **(6 marks)**

END