GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics

Module P5

Paper E

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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P5 Paper E – Marking Guide

1. artanh $x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) = \ln 3 \therefore \ln\left(\frac{1+x}{1-x}\right) = 2\ln 3 = \ln 9$ — M1 A1

$\frac{1+x}{1-x} = 9 \therefore 1 + x = 9 - 9x$ — M1

$10x = 8 \therefore x = \frac{4}{5}$ — A1 **(4)**

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2. *(a)* $f'(x) = 2\cos 2x - \cosh^2 x - 2x\cosh x\sinh x$ — M1 A2

or $2\cos 2x - \cosh^2 x - x\sinh 2x$

(b) for S.P., $f'(x) = 0$

$f'(0.3) = 0.367$; $f'(0.4) = -0.131$ — M1 A1

$f'(x)$ cont. over interval, change of sign $\therefore$ root of $f'(x) = 0 \therefore$ S.P. — A1 **(6)**

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3. $t = \tan(\frac{1}{2}x) \therefore \frac{dt}{dx} = \frac{1}{2}\sec^2(\frac{1}{2}x) = \frac{1}{2}(1 + t^2)$ — M1 A1

$\int_0^{\frac{2\pi}{3}} \frac{1}{5+4\cos x}\, dx = \int_0^{\sqrt{3}} \frac{1}{5+\frac{4(1-t^2)}{1+t^2}} \times \frac{2}{1+t^2}\, dt$ — M1 A1

$= \int_0^{\sqrt{3}} \frac{2}{5(1+t^2)+4(1-t^2)}\, dt$ — M1

$= \int_0^{\sqrt{3}} \frac{2}{9+t^2}\, dt$ — A1

$= [\frac{2}{3}\arctan(\frac{t}{3})]_0^{\sqrt{3}}$ — A1

$= \frac{2}{3}[\arctan(\frac{1}{\sqrt{3}}) - \arctan 0] = \frac{2}{3}(\frac{\pi}{6} - 0) = \frac{\pi}{9} \therefore a = \frac{1}{9}$ — M1 A1 **(9)**

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4. $y = a\cosh(\frac{x}{a})$, $\frac{dy}{dx} = \sinh(\frac{x}{a})$ — B1

$A = \int_{-a}^{a} 2\pi a\cosh(\frac{x}{a})\sqrt{1+\sinh^2(\frac{x}{a})}\, dx$ — M1 A1

$= \int_{-a}^{a} 2\pi a\cosh^2(\frac{x}{a})\, dx$ — M1 A1

$= \pi a\int_{-a}^{a} 1 + \cosh(\frac{2x}{a})\, dx$ — M1

$= \pi a[x + \frac{a}{2}\sinh(\frac{2x}{a})]_{-a}^{a}$ — A1

$= \pi a[a + \frac{a}{2}\sinh 2 - \{-a + \frac{a}{2}\sinh(-2)\}]$ — M1

$= \pi a[2a + \frac{a}{2}\sinh 2 + \frac{a}{2}\sinh 2] = \pi a^2(\sinh 2 + 2)$ — A1 **(9)**

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5. *(a)* $s = 2\psi,\ \dfrac{ds}{d\psi} = 2$ M1

$\dfrac{dx}{ds} = \cos\psi\ \therefore\ \dfrac{dx}{d\psi} = \dfrac{dx}{ds}\,\dfrac{ds}{d\psi} = 2\cos\psi$ M1 A1

$\int_0^x dx = \int_0^\psi 2\cos\psi\ d\psi$ M1

$[x]_0^x = [2\sin\psi]_0^\psi\ \ \therefore x = 2\sin\psi$ A1

$\dfrac{dy}{ds} = \sin\psi\ \therefore\ \dfrac{dy}{d\psi} = \dfrac{dy}{ds}\,\dfrac{ds}{d\psi} = 2\sin\psi$

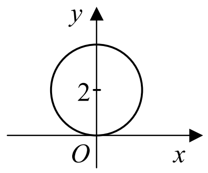
$\int_0^y dx = \int_0^\psi 2\sin\psi\ d\psi$ M1

$[y]_0^y = [-2\cos\psi]_0^\psi\ \ \therefore y = -2\cos\psi + 2$ A1

$\sin\psi = \frac{x}{2},\ \cos\psi = \frac{2-y}{2}$

$\sin^2\psi + \cos^2\psi = 1\ \therefore \frac{x^2}{4} + \frac{(2-y)^2}{4} = 1$ or $x^2 + (y-2)^2 = 4$ M1 A1

(b)



circle, centre (0, 2), radius 2 B2 **(11)**

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6. *(a)* $\cosh x\cosh y + \sinh x\sinh y$

$= \frac{1}{4}(e^x + e^{-x})(e^y + e^{-y}) + \frac{1}{4}(e^x - e^{-x})(e^y - e^{-y})$ M1

$= \frac{1}{4}(e^{x+y} + e^{x-y} + e^{y-x} + e^{-x-y} + e^{x+y} - e^{x-y} - e^{y-x} + e^{-x-y})$ A1

$= \frac{1}{2}(e^{x+y} + e^{-(x+y)})$ M1

$= \cosh(x + y)$ A1

(b) $5\cosh x + 4\sinh x \equiv R\cosh x\cosh\alpha + R\sinh x\sinh\alpha$

$\therefore\ 5 = R\cosh\alpha,\ 4 = R\sinh\alpha$ M1

$R = \sqrt{(5^2 - 4^2)} = 3$ M1 A1

(c) $\tanh\alpha = \frac{4}{5}$ M1

$\alpha = \text{artanh}\,(\frac{4}{5}) = \frac{1}{2}\ln\left(\dfrac{1+\frac{4}{5}}{1-\frac{4}{5}}\right) = \frac{1}{2}\ln 9 = \ln 3$ M1 A1

(d) $5\cosh x + 4\sinh x = 3\cosh(x + \alpha)\ \therefore$ min. value = 3 A1 **(11)**

7.	*(a)*	$u = x^{n-1},\ u' = (n-1)x^{n-2};\ v' = x\,e^{x^2},\ v = \frac{1}{2}e^{x^2}$	M1 A1	
		$I_n = [\frac{1}{2}x^{n-1}e^{x^2}]_0^1 - \int_0^1 \frac{1}{2}(n-1)x^{n-2}e^{x^2}\ dx$	A1	
		$I_n = \frac{1}{2}e - 0 - \frac{1}{2}(n-1)\int_0^1 x^{n-2}e^{x^2}\ dx$	M1	
		$I_n = \frac{1}{2}e - \frac{1}{2}(n-1)I_{n-2}$	A1	
	(b)	$I_1 = \int_0^1 x\,e^{x^2}\ dx = [\frac{1}{2}e^{x^2}]_0^1 = \frac{1}{2}e - \frac{1}{2}$	M1 A1	
		$I_3 = \frac{1}{2}e - \frac{1}{2}\times 2I_1 = \frac{1}{2}e - (\frac{1}{2}e - \frac{1}{2}) = \frac{1}{2}$	M1 A1	
		$I_5 = \frac{1}{2}e - \frac{1}{2}\times 4I_3 = \frac{1}{2}e - 2(\frac{1}{2}) = \frac{1}{2}e - 1$	M1 A1	**(11)**
8.	*(a)*	$(mx + c)^2 = 8x$	M1	
		$m^2x^2 + 2mcx + c^2 = 8x$		
		$m^2x^2 + x(2mc - 8) + c^2 = 0$	A1	
		tangent $\therefore$ "$b^2 - 4ac$" $= 0$ so $(2mc - 8)^2 - 4m^2c^2 = 0$	M1	
		$(mc - 4)^2 - m^2c^2 = 0$		
		$m^2c^2 - 8mc + 16 - m^2c^2 = 0$	M1	
		$8mc = 16\ \therefore\ mc = 2$	A1	
	(b)	$mc = 2\ \therefore\ y = mx + \frac{2}{m}$ is tangent to $x^2 + y^2 = 2$	M1	
		$x^2 + (mx + \frac{2}{m})^2 = 2$	M1	
		$x^2 + m^2x^2 + 4x + \frac{4}{m^2} = 2$		
		$x^2(1 + m^2) + 4x + (\frac{4}{m^2} - 2) = 0$	A1	
		"$b^2 - 4ac$" $= 0\ \therefore\ 16 - 4(1 + m^2)(\frac{4}{m^2} - 2) = 0$	M1	
		$2 - (1 + m^2)(\frac{2}{m^2} - 1) = 0$		
		$2 - (\frac{2}{m^2} - 1 + 2 - m^2) = 0$		
		$m^2 + 1 - \frac{2}{m^2} = 0$	A1	
		$m^4 + m^2 - 2 = 0$	M1	
		$(m^2 - 1)(m^2 + 2) = 0$		
		$\therefore\ m^2 = -2$ (no solns) or $m = \pm 1$	A1	
		$m = 1, c = 2;\ m = -1, c = -2\ \therefore\ y = x + 2$ and $y = -x - 2$	M1 A1	**(14)**
			Total	**(75)**

Performance Record – P5 Paper E

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	eqn. in inv. hyp. fns.	diff. hyp. fns.	integr. $t=\tan\frac{1}{2}x$	surface area	intrinsic coords.	hyp. idents.	reduction formula	parabola, tangent	
Marks	4	6	9	9	11	11	11	14	75
Student									