GCE Examinations

Pure Mathematics Module P5

Advanced Subsidiary / Advanced Level

Paper E

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Rosemary Smith & Shaun Armstrong

1. A student without a calculator must find the value of x given that $\operatorname{artanh} x = \ln 3$.

With clear working, show how the student could find x and state the value he should obtain.

(4 marks)

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2. $$f(x) = \sin 2x - x\cosh^2 x.$$

(a) Find $f'(x)$. **(3 marks)**

(b) Show that the curve with equation $y = f(x)$ has a stationary point in the interval $0.3 < x < 0.4$.

(3 marks)

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3. Given that

$$\int_0^{\frac{2\pi}{3}} \frac{1}{5+4\cos x}\,\mathrm{d}x = a\pi, \qquad a \in \mathbb{Q},$$

use the substitution $t = \tan(\frac{1}{2}x)$ to find the value of a. **(9 marks)**

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4. The curve C has equation $y = a\cosh\left(\frac{x}{a}\right)$, where a is a positive constant.

The area bounded by the curve C, the x-axis and the lines $x = -a$ and $x = a$ is rotated through 2π radians about the x-axis.

Show that the curved surface area of the solid generated is $\pi a^2(\sinh 2 + 2)$. **(9 marks)**

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5. The intrinsic equation of the curve C is $s = 2\psi$.

Given that s is measured from the origin,

(a) find a Cartesian equation of C, **(9 marks)**

(b) sketch C. **(2 marks)**

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6. *(a)* Using the definitions of hyperbolic functions in terms of exponential functions, prove that

$$\cosh(x + y) \equiv \cosh x \cosh y + \sinh x \sinh y.$$ **(4 marks)**

Given that

$$5\cosh x + 4\sinh x \equiv R\cosh(x + \alpha),$$

find

(b) the value of R, **(3 marks)**

(c) the value of α, giving your answer in terms of natural logarithms. **(3 marks)**

(d) Hence, or otherwise, state the minimum value of $5\cosh x + 4\sinh x$. **(1 mark)**

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7. $$I_n = \int_0^1 x^n e^{x^2}\, dx, \quad n \geq 0.$$

(a) Show that

$$I_n = \tfrac{1}{2}e - \tfrac{1}{2}(n-1)I_{n-2}, \quad n \geq 2.$$ **(5 marks)**

(b) Hence find

$$I_n = \int_0^1 x^5 e^{x^2}\, dx,$$

giving your answer in terms of e. **(6 marks)**

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8. The line with equation $y = mx + c$ is a tangent to the parabola with equation $y^2 = 8x$.

(a) Show that $mc = 2$. **(5 marks)**

The lines l_1 and l_2 are tangents to both the parabola with equation $y^2 = 8x$ and the circle with equation $x^2 + y^2 = 2$.

(b) Find the equations of l_1 and l_2. **(9 marks)**

END