GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics
Module P5

Paper D

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Written by Rosemary Smith & Shaun Armstrong

P5 Paper D – Marking Guide

1.	*(a)*	$\dfrac{dy}{dx} = \dfrac{-(x^2+1)\text{cosech}\, x \coth x - 2x \text{ cosech}\, x}{(x^2+1)^2}$	M2 A2	
	(b)	$x = 0.5,\ \dfrac{dy}{dx} = -4.55$ (2dp)	A1	**(5)**

2.	$\rho = \dfrac{ds}{d\psi} = 2(s+a)\ \therefore \int \frac{1}{s+a}\ ds = \int 2\ d\psi$	M1 A1	
	$\ln\lvert s+a \rvert = 2\psi + c$	A1	
	$s + a = e^{2\psi + c} = e^{2\psi} \times e^{c}$	M1	
	$\therefore\ s = Ae^{2\psi} - a$ [where $A = e^{c}$]	A1	**(5)**

3.	*(a)*	$\sinh 3x \equiv \sinh(2x + x)$	M1	
		$\equiv \sinh 2x \cosh x + \cosh 2x \sinh x$	A1	
		$\equiv 2 \sinh x \cosh^2 x + (1 + 2\sinh^2 x)\sinh x$	M1	
		$\equiv 2 \sinh x(1 + \sinh^2 x) + \sinh x + 2\sinh^3 x$	M1	
		$\equiv 2 \sinh x + 2\sinh^3 x + \sinh x + 2\sinh^3 x$		
		$\equiv 4\sinh^3 x + 3\sinh x$	A1	
	(b)	$4\sinh^3 x + 3\sinh x = 7\sinh^2 x$		
		$\sinh x(4\sinh^2 x - 7\sinh x + 3) = 0$	M1	
		$\sinh x(4\sinh x - 3)(\sinh x - 1) = 0$	M1	
		$\sinh x = 0$ or $\frac{3}{4}$ or 1	A1	
		$x = 0$ or $\ln\left(\frac{3}{4} + \sqrt{1 + \frac{9}{16}}\right)$ or $\ln\left(1 + \sqrt{1+1}\right)$	M1	
		$x = 0$ or $\ln 2$ or $\ln(1 + \sqrt{2})$	A2	**(11)**

---

4. *(a)* $\int \frac{1}{\sqrt{9-4x^2}}\,dx = \int \frac{1}{2}\frac{1}{\sqrt{\frac{9}{4}-x^2}}\,dx$ M1

$= \frac{1}{2}\arcsin\left(\frac{2x}{3}\right) + c$ M1 A1

(b) $\int \frac{1-2x}{\sqrt{9-4x^2}}\,dx = \frac{1}{2}\arcsin\left(\frac{2x}{3}\right) - \int \frac{2x}{\sqrt{9-4x^2}}\,dx$ M1

$= \frac{1}{2}\arcsin\left(\frac{2x}{3}\right) + \frac{1}{2}\sqrt{9-4x^2} + c$ M1 A1

(c) $\int_1^y \frac{1}{y}\,dy = \int_0^x \frac{1-2x}{\sqrt{9-4x^2}}\,dx$ M1 A1

$[\ln|y|]_1^y = [\frac{1}{2}\arcsin\left(\frac{2x}{3}\right) + \frac{1}{2}\sqrt{9-4x^2}\,]_0^x$ A1

$\ln|y| - \ln 1 = \frac{1}{2}\arcsin\left(\frac{2x}{3}\right) + \frac{1}{2}\sqrt{9-4x^2} - (\frac{1}{2}\arcsin 0 + \frac{3}{2})$ M1 A1

$\ln|y| = \frac{1}{2}\arcsin\left(\frac{2x}{3}\right) + \frac{1}{2}\sqrt{9-4x^2} - \frac{3}{2}$ A1 **(12)**

---

5. *(a)* $2y\frac{dy}{dx} = 4a \;\therefore\; \frac{dy}{dx} = \frac{2a}{y}$ M1

at P, $\frac{dy}{dx} = \frac{2a}{2ap} = \frac{1}{p}$ A1

eqn. is $y - 2ap = \frac{1}{p}(x - ap^2)$ M1

giving $yp = x + ap^2$ A1

(b) grad of $PQ = \frac{2ap-2aq}{ap^2-aq^2} = \frac{2(p-q)}{(p+q)(p-q)} = \frac{2}{p+q}$ M1 A1

grad of $PS = \frac{2ap-0}{ap^2-a} = \frac{2p}{p^2-1}$ A1

$\therefore \frac{2}{p+q} = \frac{2p}{p^2-1}$ M1

$p^2 - 1 = p(p+q)$

$p^2 - 1 = p^2 + pq$

$\therefore pq = -1$ A1

(c) tangent at P: $yp = x + ap^2$ (i)

tangent at Q: $yq = x + aq^2$ (ii)

(i) $\times q$: $ypq = xq + ap^2q$

(ii) $\times p$: $ypp = xp + apq^2$ M1

subtracting $0 = x(p-q) + apq(q-p)$ M1

$0 = x - apq$ A1

$pq = -1 \;\therefore\; x = -a \;\therefore$ meet on directrix A1 **(13)**

6.	*(a)*	$u = \sec^{n-2}x,\ u' = (n-2)\sec^{n-3}x\sec x\tan x;\ \ v' = \sec^2 x,\ v = \tan x$	M1 A1	
		$I_n = [\sec^{n-2}x\tan x]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} (n-2)\sec^{n-2}x\tan^2 x\ \mathrm{d}x$	A1	
		$I_n = (\sqrt{2})^{n-2} - 0 - (n-2)\int_0^{\frac{\pi}{4}} \sec^{n-2}x(\sec^2 x - 1)\ \mathrm{d}x$	M1 A1	
		$I_n = (\sqrt{2})^{n-2} - (n-2)\int_0^{\frac{\pi}{4}} \sec^n x\ \mathrm{d}x + (n-2)\int_0^{\frac{\pi}{4}} \sec^{n-2}x\ \mathrm{d}x$		
		$I_n = (\sqrt{2})^{n-2} - (n-2)I_n + (n-2)I_{n-2}$	M1	
		$(n-1)I_n = (\sqrt{2})^{n-2} + (n-2)I_{n-2}$	A1	
	(b)	$I_1 = \int_0^{\frac{\pi}{4}} \sec x\ \mathrm{d}x = [\ln\lvert\sec x + \tan x\rvert]_0^{\frac{\pi}{4}}$	M1	
		$= \ln(\sqrt{2}+1) - \ln(1+0) = \ln(\sqrt{2}+1)$	M1 A1	
		$2I_3 = (\sqrt{2})^1 + I_1 = \sqrt{2} + \ln(\sqrt{2}+1)$	M1 A1	
		$I_3 = \frac{1}{2}\sqrt{2} + \frac{1}{2}\ln(\sqrt{2}+1)$	A1	**(13)**
7.	*(a)*	$x^2 = a^2\sinh^2 u,\ \ x = a\sinh u,\ \ \frac{\mathrm{d}x}{\mathrm{d}u} = a\cosh u$	M1 A1	
		$\int \sqrt{a^2+x^2}\ \mathrm{d}x = \int \sqrt{a^2 + a^2\sinh^2 u}\,(a\cosh u)\ \mathrm{d}u$	M1	
		$= \int a^2\cosh^2 u\ \mathrm{d}u$	A1	
		$= \frac{1}{2}a^2\int \cosh 2u + 1\ \mathrm{d}u$	M1	
		$= \frac{1}{2}a^2[\frac{1}{2}\sinh 2u + u] + c$	A1	
		$= \frac{1}{2}a^2\sinh u\cosh u + \frac{1}{2}a^2 u + c$	M1	
		$= \frac{1}{2}a^2\times\frac{x}{a}\times\sqrt{1+\frac{x^2}{a^2}} + \frac{1}{2}a^2\operatorname{arsinh}(\frac{x}{a}) + c$	M1	
		$= \frac{1}{2}ax\sqrt{1+\frac{x^2}{a^2}} + \frac{1}{2}a^2\operatorname{arsinh}(\frac{x}{a}) + c$		
		$= \frac{1}{2}x\sqrt{a^2+x^2} + \frac{1}{2}a^2\operatorname{arsinh}(\frac{x}{a}) + c$	A1	
	(b)	$x = 2t,\ \frac{\mathrm{d}x}{\mathrm{d}t} = 2;\ \ y = t^2,\ \frac{\mathrm{d}y}{\mathrm{d}t} = 2t$	M1	
		$s = \int_0^3 \sqrt{4+4t^2}\ \mathrm{d}t$	M1 A1	
		$s = 2\int_0^3 \sqrt{1+t^2}\ \mathrm{d}t$	A1	
	(c)	$s = 2[\frac{t}{2}\sqrt{1+t^2} + \frac{1}{2}\operatorname{arsinh} t]_0^3$	M1	
		$s = 2[(\frac{3}{2}\sqrt{10} + \frac{1}{2}\operatorname{arsinh} 3) - (0+0)] = 3\sqrt{10} + \operatorname{arsinh} 3$	M1 A1	**(16)**
			Total	**(75)**

Performance Record – P5 Paper D

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	diff. hyp. fns	intrinsic coords	eqn. in hyp. fns.	integr. std. forms	parabola, tangent	reduction formula	integr. using hyp. sub., arc length	
Marks	5	5	11	12	13	13	16	75
Student								