GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics
Module P5

Paper B

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

Written by Rosemary Smith & Shaun Armstrong

© *Solomon Press*

These sheets may be copied for use solely by the purchaser's institute.

P5 Paper B – Marking Guide

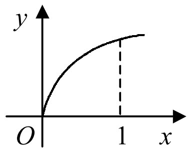
1.	$\dfrac{-y}{\sqrt{1-x^2}} + \arccos x \dfrac{dy}{dx} - \frac{1}{\pi}e^{2x} - \frac{2x}{\pi}e^{2x} = 0$	M1 A2	
	when $x = 0$, $y \times \frac{\pi}{2} - 0 - 1 = 0 \therefore y = \frac{2}{\pi}$	B1	
	when $x = 0$, $-\frac{2}{\pi} + \frac{\pi}{2}\dfrac{dy}{dx} - \frac{1}{\pi} - 0 = 0$	M1 A1	
	$\therefore \frac{\pi}{2}\dfrac{dy}{dx} = \frac{3}{\pi}$ so $\dfrac{dy}{dx} = \dfrac{6}{\pi^2}$	A1	**(7)**

2.	$f'(x) = 5\sinh x + 3\cosh x$	M1	
	S.P. $\therefore 5\sinh x + 3\cosh x = 0$ giving $\tanh x = -\frac{3}{5}$	M1 A1	
	$x = \text{artanh}\,(-\frac{3}{5}) = \frac{1}{2}\ln\left(\dfrac{1-\frac{3}{5}}{1+\frac{3}{5}}\right)$	M1 A1	
	$x = \frac{1}{2}\ln\frac{1}{4} = -\ln 2$	A1	
	$f(-\ln 2) = 5\cosh(-\ln 2) + 3\sinh(-\ln 2) = 4$	M1	
	$\therefore p = -1, q = 2, r = 4$	A1	**(8)**

3.	*(a)*	$x(mx + c) = -9 \therefore mx^2 + cx + 9 = 0$	M1 A1	
		tangent $\therefore$ "$b^2 - 4ac$" $= 0 \therefore c^2 - 4 \times m \times 9 = 0$	M1	
		$\therefore c^2 = 36m$ giving $c = \pm 6\sqrt{m}$	A1	
	(b)	$(4, -2) \therefore -2 = 4m + c$ and $c^2 = 36m$		
		$\therefore (-2 - 4m)^2 = 36m$	M1	
		$4 + 16m + 16m^2 = 36m$		
		giving $4m^2 - 5m + 1 = 0$		
		$(4m - 1)(m - 1) = 0$	M1	
		$m = \frac{1}{4}$ or 1	A1	
		if $m = \frac{1}{4}$, $c = -3$; if $m = 1$, $c = -6$	M1	
		$\therefore$ tangents are $y = \frac{1}{4}x - 3$ and $y = x - 6$	A1	**(9)**

© *Solomon Press*

4. $y^2 = x \;\therefore\; 2y\frac{dy}{dx} = 1$ so $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$ — M1 A1



$\therefore$ lower limit = 0 — M1

$A = \int_0^1 2\pi y\sqrt{1+\frac{1}{4x}}\ dx$ — M1 A1

$= \int_0^1 2\pi\sqrt{x}\sqrt{1+\frac{1}{4x}}\ dx = \int_0^1 \pi\sqrt{4x+1}\ dx$ — M1 A1

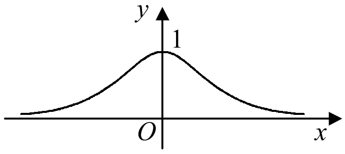
$= \pi[\frac{2}{3}\times\frac{1}{4}(4x+1)^{\frac{3}{2}}]_0^1$ — M1 A1

$= \frac{1}{6}\pi[5^{\frac{3}{2}} - 1^{\frac{3}{2}}] = \frac{1}{6}\pi(5\sqrt{5} - 1)$ — M1 A1 **(11)**

---

5. *(a)* $\cosh x = \frac{1}{2}(e^x + e^{-x}) \;\therefore\; \text{sech}\, x = \frac{2}{e^x + e^{-x}}$ — B1

(b)



B2

(c) $\int \text{sech}\, x\ dx = \int \frac{2}{e^x + e^{-x}}\ dx = \int \frac{2e^x}{e^{2x}+1}\ dx$

$u = e^x \;\therefore\; \frac{du}{dx} = e^x$ — M1

$I = \int \frac{2}{u^2+1}\ du$ — A1

$= 2\arctan u + c = 2\arctan e^x + c$ — M1 A1

(d) $V = \int_{-a}^{a} \pi\,\text{sech}^2 x\ dx$ — M1

$= [\pi\tanh x]_{-a}^{a}$ — A1

$= \pi[\tanh a - \tanh(-a)] = 2\pi\tanh a$ — M1 A1

(e) as $a \to \infty$, $\tanh a \to 1$, $V \to 2\pi$ $\therefore$ limit of volume is 2π — A1 **(12)**

© *Solomon Press*

6.	*(a)*	$u = (2 - x^2)^n,\ u' = -2nx(2 - x^2)^{n-1};\ \ v' = 1,\ v = x$	M1	
		$I_n = [x(2 - x^2)^n]_0^{\sqrt{2}} - \int_0^{\sqrt{2}} -2nx^2(2 - x^2)^{n-1}\ \mathrm{d}x$	A1	
		$I_n = [0 - 0] - 2n\int_0^{\sqrt{2}} (-x^2 + 2 - 2)(2 - x^2)^{n-1}\ \mathrm{d}x$	M2 A1	
		$I_n = -2n\int_0^{\sqrt{2}} (2 - x^2)(2 - x^2)^{n-1}\ \mathrm{d}x\ - 2n\int_0^{\sqrt{2}} -2(2 - x^2)^{n-1}\ \mathrm{d}x$	M1	
		$I_n = -2n\int_0^{\sqrt{2}} (2 - x^2)^n\ \mathrm{d}x\ + 4n\int_0^{\sqrt{2}} (2 - x^2)^{n-1}\ \mathrm{d}x$	A1	
		$I_n = -2nI_n + 4nI_{n-1}$	M1	
		$(1 + 2n)I_n = 4nI_{n-1}$		
		$I_n = \frac{4n}{2n+1} I_{n-1}$	A1	
	(b)	$I_0 = \int_0^{\sqrt{2}} \mathrm{d}x\ =\ [x]_0^{\sqrt{2}} = \sqrt{2}$	B1	
		$I_1 = \frac{4}{3} I_0 = \frac{4}{3}\sqrt{2}$	M1	
		$I_2 = \frac{8}{5} I_1 = \frac{8}{5} \times \frac{4}{3}\sqrt{2}$	M1	
		$I_3 = \frac{12}{7} I_2 = \frac{12}{7} \times \frac{8}{5} \times \frac{4}{3}\sqrt{2} = \frac{128}{35}\sqrt{2}$	A1	**(13)**
7.	*(a)*	$\rho\ =\ \frac{\mathrm{d}s}{\mathrm{d}\psi} = \frac{\frac{1}{2}\sec^2 \frac{1}{2}\psi}{\tan \frac{1}{2}\psi}$	M1 A1	
		$= \frac{1}{2} \times \frac{1}{\cos^2 \frac{1}{2}\psi} \times \frac{\cos \frac{1}{2}\psi}{\sin \frac{1}{2}\psi} = \frac{1}{2\cos \frac{1}{2}\psi \sin \frac{1}{2}\psi} = \frac{1}{\sin\psi} = \operatorname{cosec}\psi$	M1 A1	
	(b)	$\frac{\mathrm{d}s}{\mathrm{d}\psi} = \operatorname{cosec}\psi,\ \ \frac{\mathrm{d}y}{\mathrm{d}s} = \sin\ \psi$		
		$\frac{\mathrm{d}y}{\mathrm{d}\psi} = \frac{\mathrm{d}y}{\mathrm{d}s}\ \frac{\mathrm{d}s}{\mathrm{d}\psi} = 1\ \ \therefore\ y = \psi + c$	M1 A1	
		$y = \frac{\pi}{2},\ \psi = \frac{\pi}{2}\ \ \ \therefore\ c = 0\ \text{ so }\ y = \psi$	M1 A1	
	(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \tan\ \psi\ = \tan y$	M1	
		$\int_{\frac{\pi}{2}}^{y} \cot y\ \ \mathrm{d}y = \int_0^x \mathrm{d}x$	M1 A1	
		$[\ln \lvert \sin y \rvert]_{\frac{\pi}{2}}^{y} = [x]_0^x$	M1 A1	
		$\ln (\sin y) - \ln 1 = x - 0\ \ \ [0 < y \le \frac{\pi}{2}\ \ \therefore\ \sin y > 0]$	M1	
		$x = \ln (\sin y)$	A1	**(15)**
			Total	**(75)**

© *Solomon Press*

Performance Record – P5 Paper B

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	diff. inv. trig.	diff. hyp. fns., eqn. in hyp. fns.	hyperbola, tangent	surface area	integr. hyp. fns	reduction formula	intrinsic coords, rad. of curv.	
Marks	7	8	9	11	12	13	15	75
Student								

© *Solomon Press*