GCE Examinations

Pure Mathematics Module P5

Advanced Subsidiary / Advanced Level

Paper B

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Rosemary Smith & Shaun Armstrong

1. Given that

$$y \arccos x - \frac{x}{\pi} e^{2x} - 1 = 0,$$

find the value of $\frac{dy}{dx}$ at the point where $x = 0$, giving your answer in terms of π. **(7 marks)**

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2. $f(x) = 5\cosh x + 3\sinh x$.

The minimum value of $f(x)$ occurs at the point $(p \ln q, r)$ where p, q and r are integers.

Find the values of p, q and r. **(8 marks)**

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3. The line $y = mx + c$ is a tangent to the rectangular hyperbola with equation $xy = -9$.

(a) Show that $c = \pm 6\sqrt{m}$. **(4 marks)**

(b) Hence, or otherwise, find the equations of the tangents from the point $(4, -2)$ to the rectangular hyperbola $xy = -9$. **(5 marks)**

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4. The curve C is defined by

$$y^2 = x, \qquad x \geq 0, \ y \geq 0.$$

The region between C, the x-axis and the line $x = 1$ is rotated through 2π about the x-axis.

Show that the area of the surface generated is

$$\frac{\pi}{6}(5\sqrt{5} - 1).$$ **(11 marks)**

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5. *(a)* Using the definition of $\cosh x$ in terms of exponential functions, express $\operatorname{sech} x$ in terms of e^x and e^{-x}.

(1 mark)

(b) Sketch the graph of $y = \operatorname{sech} x$. **(2 marks)**

(c) Show that $\int \operatorname{sech} x \ \mathrm{d}x = 2 \arctan e^x + c$. **(4 marks)**

The curve C has equation $y = \operatorname{sech} x$. The region between C, the x-axis and the lines $x = -a$ and $x = a$, where a is a positive constant, is rotated through 2π about the x-axis.

(d) Find the volume of revolution of the solid generated. **(4 marks)**

(e) Find the limit of the volume of revolution as $a \to \infty$. **(1 mark)**

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6. $$I_n = \int_0^{\sqrt{2}} (2 - x^2)^n \ \mathrm{d}x, \quad n \geq 0.$$

(a) Show that

$$I_n = \frac{4n}{2n+1} I_{n-1}, \quad n \geq 1.$$ **(9 marks)**

(b) Hence evaluate I_3, leaving your answer in surd form. **(4 marks)**

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7. The curve C has intrinsic equation

$$s = \ln(\tan \tfrac{1}{2}\psi), \quad 0 < \psi \leq \tfrac{\pi}{2}.$$

(a) Show that radius of curvature of C is given by $\rho = \operatorname{cosec} \psi$. **(4 marks)**

Given that $y = \psi = \frac{\pi}{2}$ when $x = 0$,

(b) show that $y = \psi$, **(4 marks)**

(c) use integration to show that a Cartesian equation of C is $x = \ln(\sin y)$. **(7 marks)**

END