

GCE Examinations

Pure Mathematics

Module P5

Advanced Subsidiary / Advanced Level

Paper A

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working will gain no credit.



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1. A curve has the equation

$$y = x + 2x^2 + 5x^3.$$

Show that the radius of curvature of the curve at the origin is $\frac{1}{\sqrt{2}}$. **(5 marks)**

2. Show that

$$\int_0^{\ln 2} x \operatorname{sech}^2 x \, dx = \frac{3}{5} \ln 2 - \ln\left(\frac{5}{4}\right). \quad \text{(8 marks)}$$

3. (a) Prove that

$$\frac{d}{dx} (\arcsin 2x) = \frac{2}{\sqrt{1-4x^2}}. \quad \text{(3 marks)}$$

Given that

$$f(x) = 2x \arcsin 2x + \sqrt{1-4x^2},$$

- (b) show that

$$f''(x)[f(x) - xf'(x)] = 4. \quad \text{(6 marks)}$$

4.

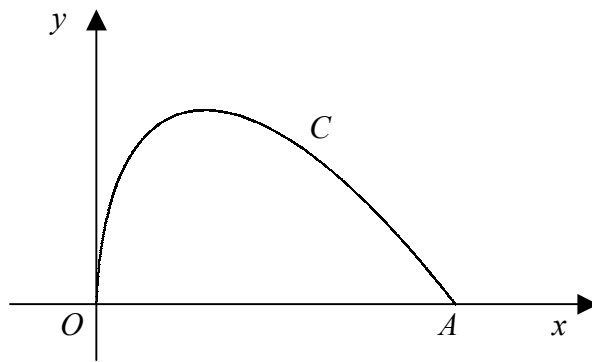


Fig. 1

The parametric coordinates of the curve C shown in Figure 1 are

$$x = t^2, \quad y = t - \frac{1}{3}t^3, \quad 0 \leq t \leq a.$$

The curve C meets the x -axis at the point A where $t = a$.

(a) Find the value of a . **(2 marks)**

The curve C is rotated through 2π about Ox .

(b) Find the surface area of the solid generated. **(8 marks)**

5. (a) Using the definitions of $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x} , prove that

$$\cosh 2x = 2 \cosh^2 x - 1. \quad \text{(3 marks)}$$

(b) Solve the equation

$$2 \cosh 2x = 13 \cosh x - 12,$$

giving your answers in terms of natural logarithms. **(7 marks)**

Turn over

6. $x^2 - 10x + 41 \equiv (x + a)^2 + b.$

(a) Find the values of the constants a and b . **(2 marks)**

(b) Show that

$$\int_5^9 \frac{x}{\sqrt{x^2 - 10x + 41}} dx = p(\sqrt{2} - 1) + q \ln r,$$

stating your values of p , q and r . **(8 marks)**

7. $I_n = \int_0^{\frac{\pi}{2}} x^n \cos x dx, \quad n \geq 0.$

(a) Prove that

$$I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2} \quad n \geq 2. \quad \textbf{(5 marks)}$$

(b) Hence find the value of I_4 , giving your answer in terms of π . **(6 marks)**

8. The rectangular hyperbola C has equation $xy = c^2$, where c is a positive constant.

(a) Show that an equation of the tangent to C at the point $P\left(cp, \frac{c}{p}\right)$ is

$$x + yp^2 = 2cp. \quad \textbf{(4 marks)}$$

The tangent to C at P meets the x -axis at the point X .

The point Q on C has coordinates $\left(cq, \frac{c}{q}\right)$, $q \neq p$ such that QX is parallel to the y -axis.

(b) Show that $q = 2p$. **(3 marks)**

M is the mid-point of PQ .

(c) Find, in Cartesian form, an equation of the locus of M as p varies. **(5 marks)**

END