1. Given that $\cosh u = 2$, find, in surd form, the possible values of $\sinh u$. **(3 marks)**

2. Find the radius of curvature of the curve with intrinsic equation $s = \cosh^2 \psi$ at the point where $\psi = \frac{1}{2}\ln 3$. **(5 marks)**

3. Find the values of m for which the line $y = mx + 1$ is a tangent to the hyperbola with equation
$$\frac{x^2}{4} - \frac{y^2}{3} = 1.$$
(6 marks)

4. Given that $y = (\text{arsinh } x)^2$,
   (a) show that $(x^2 + 1)\left(\frac{dy}{dx}\right)^2 = cy$, where c is a constant to be found. **(4 marks)**
   (b) Find an expression for $\frac{d^2y}{dx^2}$ in terms of x and $\frac{dy}{dx}$. **(3 marks)**

5. (a) Express in partial fractions : $\frac{t}{(t+1)^2(t^2+1)}$. **(8 marks)**
   (b) Hence show that $\int \frac{t}{(t+1)^2(t^2+1)}\, dt = k(\arctan t + \frac{1}{t+1})$, where k is a rational number to be found. **(3 marks)**

6. Given that $I_n = \int x^n\sqrt{(1-x)}\, dx$, where n is a positive integer,
   (a) prove that for $n \geq 1$, $(2n + 3)\, I_n = 2(nI_{n-1} - x^n(1-x)^{3/2})$ **(9 marks)**
   (b) Find I_0 in terms of x and hence evaluate
$$\int_{-3}^{0} x\sqrt{(1-x)}\, dx.$$
(5 marks)

7. The points P and Q lie on the rectangular hyperbola with equation $xy = 16$.
At P, $x = 4p$ and at Q, $x = 8p$.
(a) Write down the y-coordinates of P and Q. **(2 marks)**
(b) Show that the straight line l through P and Q has equation $2p^2y + x = 12p$. **(4 marks)**

The line l cuts the x-axis at A and the y-axis at B.
(c) Find the coordinates of the mid-point of AB. **(5 marks)**
(d) Find an equation of the locus of M, and state what type of curve this locus is. **(3 marks)**

8. The arc of the curve $y = \cos x$ between $x = 0$ and $x = \frac{\pi}{2}$ is rotated once about the x-axis.
(a) Show that the area of the surface formed is equal to $2\pi \int_0^1 \sqrt{(1 + u^2)}\, du$. **(9 marks)**
(b) Using the substitution $u = \sinh t$, evaluate this integral. **(6 marks)**

