1. Starting from the definitions of sinh x and cosh x in terms of natural logarithms, prove that $1 + 2\sinh^2 x \equiv \cosh 2x$. **(4 marks)**

2. Given that $f(x) = \dfrac{1}{x^2 + 2x + 50}$,

   (a) find $\int f(x)\,dx$. **(3 marks)**

   (b) Hence find the area of the region bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = -1$ and $x = 6$, giving your answer in an exact form. **(2 marks)**

3. A curve is given by the parametric equations

$$x = 2\sinh^3 t, \qquad y = 3\cosh^2 t,$$

   for $0 \leq t \leq \ln 3$.

   Find, in terms of e, the total length of the curve. **(8 marks)**

4. An ellipse passes through the point (4, 2) and has its foci at (−2, 0) and (2, 0). The equations of its directrices are $x = 4$ and $x = -4$.

   (a) Obtain the equation of the ellipse in the form $px^2 + qy^2 = r$, where p, q and r are integers to be found. **(5 marks)**

   (b) Find the radius of curvature of the ellipse at the point (4, 2). **(5 marks)**

5. $I_n(x)$ is defined to be $\int_0^x \sec^n t\,dt$, where $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

   (a) Obtain a reduction formula for $I_n(x)$ in terms of $I_{n-2}(x)$. **(8 marks)**

   (b) Show that $\int_0^{\pi/4} \sec^2 t\,dt = 1$ and hence or otherwise evaluate $\int_0^{\pi/4} \sec^4 t\,dt$. **(5 marks)**

6. (a) Sketch, for $-1 < x < 1$, the curve C with equation

$$y = x + \frac{1}{\sqrt{1-x^2}}.$$ **(3 marks)**

(b) Find the area of the region bounded by C, the x-axis and the lines $x = -\frac{1}{2}$ and $x = \frac{1}{2}$. **(5 marks)**

(c) Show that $\frac{d}{dx}(\sqrt{(1-x^2)}) = \frac{-x}{\sqrt{1-x^2}}$. **(2 marks)**

(d) Calculate, in terms of π, the volume of the solid formed when the area in part (b) is rotated once completely about the x-axis. **(7 marks)**

7. (a) Show that the normal to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ has equation

$$px + y = ap(p^2 + 2).$$ **(5 marks)**

(b) If this normal meets the parabola again at $Q(aq^2, 2aq)$, show that

$$q = -p - \frac{2}{p}.$$ **(6 marks)**

(c) Find an equation of the locus of the mid-point of PQ as P varies. **(7 marks)**

