1. Given that $y = \arctan x$, prove that $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$ **(3 marks)**

2. Given that $y = \sinh x$,

   (a) find, in terms of natural logarithms, the value of x for which $7y = 24$. **(3 marks)**

   (b) For this value of x, find the exact value of $\sinh 2x$. **(3 marks)**

3. Given that $I_n = \int_0^{\pi} \sin^n x \, dx$,

   (a) show that, for $n \geq 1$, $I_n = \dfrac{n-1}{n} I_{n-2}$. **(6 marks)**

   (b) Hence find the exact value of I_5. **(4 marks)**

4. A curve C has parametric equations $x = 3t^2$, $y = 6t$.

   (a) Give the name for the type of curve of which C is an example. **(1 mark)**

   (b) Find the radius of curvature of C at the point $(3, -6)$. **(4 marks)**

   (c) Find the value of p for which the the tangent to C at $(3p^2, 6p)$ passes through the point $(0, 1)$. **(4 marks)**

5. (a) Find

   (i) $\displaystyle\int \frac{1}{\sqrt{x^2+8x+20}} \, dx$, (ii) $\displaystyle\int \frac{1}{x^2+8x+20} \, dx$. **(6 marks)**

   (b) Show that $\displaystyle\int_{-6}^{-2} \frac{4}{x^2+8x+20} \, dx = \pi$. **(3 marks)**

6. (a) Show that the length of the arc of the curve $y = \frac{2}{3}x^{3/2}$ between the points where $x = 0$ and $x = 3$ is equal to

$$\int_0^3 \sqrt{(1+x)} \, dx.$$ **(4 marks)**

   (b) Using the substitution $1 + x = u^2$, or otherwise, evaluate this length. **(5 marks)**

7. (a) Sketch the curve with equation $y = \cosh x$. **(2 marks)**

(b) Show that the normal to this curve at the point P where $x = \ln 2$ cuts the x-axis at the point $(\ln 2 + \frac{15}{16}, 0)$. **(7 marks)**

The finite region bounded by the curve $y = \cosh x$, the x and y axes and the normal at P is rotated through 360° about the x-axis.

(c) Find the volume of the solid formed. **(5 marks)**

8. (a) Show that an equation of the tangent at the point $(\frac{5}{3}\cos\theta, \frac{5}{4}\sin\theta)$ to the ellipse $9x^2 + 16y^2 = 25$ is $3x\cos\theta + 4y\sin\theta = 5$. **(6 marks)**

Given that this tangent meets the x-axis at P and the y-axis at Q, and that O is the origin,

(b) show that the area of triangle OPQ is $|\, k \operatorname{cosec} 2\theta \,|$, where k is a constant to be found. **(4 marks)**

(c) Show also that as θ varies, the locus of the mid-point of PQ is the curve with equation

$$9x^2 + 16y^2 = \frac{576}{25}x^2y^2.$$ **(5 marks)**