PURE MATHS 5 (A) TEST PAPER 9 : ANSWERS AND MARK SCHEME

1.	$\sinh^2 u = \cosh^2 u - 1 = 3 \qquad \sinh u = \pm\sqrt{3}$	M1 A1 A1	3
2.	$\rho = \dfrac{ds}{d\psi} = 2\sinh\psi\cosh\psi = \sinh 2\psi$	M1 A1	
	When $\psi = \dfrac{1}{2}\ln 3, \rho = \sinh(\ln 3) = \dfrac{1}{2}\left(3 - \dfrac{1}{3}\right) = \dfrac{4}{3}$	M1 A1 A1	5
3.	Where line meets curve, $\dfrac{x}{4} - \dfrac{(mx+1)^2}{3} = 1 \qquad 3x^2 - 4(m^2x^2 + 2mx + 1) = 12$	M1 A1	
	$(3 - 4m^2)x^2 - 8mx - 16 = 0$ For one real root,	A1 M1	
	$64m^2 + 64(3 - 4m^2) = 0 \qquad 3m^2 = 3 \qquad m = \pm 1$	A1 A1	6
4.	(a) $\dfrac{dy}{dx} = 2\,\text{arsinh}\,x \,.\, \dfrac{1}{\sqrt{1+x^2}} \qquad \left(\dfrac{dy}{dx}\right)^2 = \dfrac{4y}{1+x^2} \qquad (x^2+1)\left(\dfrac{dy}{dx}\right)^2 = 4y$	M1 A1 M1 A1	
	(b) $\dfrac{d^2y}{dx^2} = \dfrac{2}{1+x^2} - \dfrac{2x}{(1+x^2)^{3/2}}\text{arsinh}\,x = \dfrac{1}{1+x^2}\left(2 - x\dfrac{dy}{dx}\right)$	M1 A1 A1	7
5.	(a) Partial fractions are of the form $\dfrac{A}{t+1} + \dfrac{B}{(t+1)^2} + \dfrac{Ct+D}{t^2+1}$	B1	
	$A(t+1)(t^2+1) + B(t^2+1) + (Ct+D)(t+1)^2 = t$	M1 A1	
	Let $t = -1$: $B = -1/2 \qquad A + C = 0,\ A + B + C + D = 0,$	A1 M1	
	$A + C + 2D = 1,\ A + B + D = 0 \qquad A = C = 0,\ D = ½$	A1 A1	
	Expression = $\dfrac{1}{2(t^2+1)} - \dfrac{1}{2(t+1)^2}$	A1	
	(b) Integrating gives $\dfrac{1}{2}\arctan t + \dfrac{1}{2(t+1)} \qquad k = \dfrac{1}{2}$	M1 A1 A1	11
6.	(a) Let $u = x^n,\ dv = (1-x)^{1/2}\,dx \qquad du = nx^{n-1}\,dx,\ v = -\frac{2}{3}(1-x)^{3/2}$	M1 A1	
	$= -\dfrac{2}{3}x^n(1-x)^{3/2} + \dfrac{2}{3}n\int x^{n-1}(1-x)^{1/2} - x^n(1-x)^{1/2}dx$	M1 A1 A1	
	$= -\dfrac{2}{3}x^n(1-x)^{3/2} + \dfrac{2}{3}n(I_{n-1} - I_n)$	A1	
	$\left(1 + \dfrac{2n}{3}\right)I_n = -\dfrac{2}{3}x^n(1-x)^{3/2} + \dfrac{2}{3}nI_{n-1}$	M1 A1	
	$(2n+3)I_n = 2(nI_{n-1} - x^n(1-x)^{3/2})$	A1	
	(b) $[I_0]_{-3}^{0} = \left[-\dfrac{2}{3}(1-x)^{3/2}\right]_{-3}^{0} = \dfrac{14}{3}$	M1 A1	
	$[5I_1]_{-3}^{0} = 2I_0 - [2x(1-x)^{3/2}]_{-3}^{0} = 2\left(\dfrac{14}{3}\right) - [0 - (-6)(8)] = -\dfrac{116}{3}$	M1 A1	
	$[I_1]_{-3}^{0} - \dfrac{116}{15}$	A1	14

7. (a) At $P, y = \frac{4}{p}$ At $Q, y = \frac{2}{p}$ B1 B1

(b) Gradient of $PQ = \frac{2/p - 4/p}{4p} = \frac{-1}{2p^2}$ M1

Equation of PQ is $y - \frac{2}{p} = -\frac{1}{2p^2}(x - 8p)$ $2p^2y + x = 12p$ M1 A1 A1

(c) At $A, y = 0$ $x = 12p$ At $B, x = 0$ $y = \frac{6}{p}$ M1 A1 A1

Mid-point is $\left(6p, \frac{3}{p}\right)$ M1 A1

(d) Locus of M is $xy = 18$, which is another rectangular hyperbola M1 A1 A1 14

8. (a) $\frac{dy}{dx} = \sin x$ $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{1/2} = \sqrt{1 + \sin^2 x}$ B1 M1 A1

Area $= 2\pi \int_0^{\pi/2} \cos x \sqrt{1 + \sin^2 x}\, dx$ Let $u = \sin x$, $du = \cos x\, dx$ B1 M1 A1

Limits become $u = 0, 1$ Area $= 2\pi \int_0^1 \sqrt{1 + u^2}\, du$ B1 M1 A1

(b) Let $u = \sinh t$, so $du = \cosh t\, dt$ Area $= 2\pi \int_0^{ar\sinh 1} \cosh^2 t\, dt$ M1 A1 A1

$= \pi \int_0^{ar\sinh 1} \cosh 2t + 1\, dt = \pi \left[\frac{\sinh 2t}{2} + t\right]_0^{ar\sinh 1} = \pi(\text{arsinh } 1 + \sqrt{2})$ M1 A1 A1 15