PURE MATHS 5 (A) TEST PAPER 8 : ANSWERS AND MARK SCHEME

1.	$\cosh 2x = 3 - 2\cosh x$	$2\cosh^2 x - 1 = 3 - 2\cosh x$	B1	
	$\cosh^2 x + \cosh x - 2 = 0$	$(\cosh x - 1)(\cosh x + 2) = 0$	M1 A1	
	$\cosh x = 1$ or $\cosh x = -2$	$x = 0$ Point is (0, 1)	M1 A1	5
2.	(a) $y = \arccos x : x = \cos y$	$\frac{dx}{dy} = -\sin y = -\sqrt{1-x^2}$ $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$	M1 A1 A1	
	(b) $\frac{dy}{dx} = \frac{-1}{\sqrt{1-(\ln 2x)^2}}.\frac{1}{x} = -2$ when $x = \frac{1}{2}$		M1 A1 A1	6
3.	$\frac{dy}{dx} = \frac{6t^2}{6t} = t$	Arc length $s = \int_0^t (1+t^2)^{1/2}.6t\,dt$	B1 M1 A1	
	$= 6\left[\frac{1}{3}(1+t^2)^{3/2}\right]_0^t = 2(1+t^2)^{3/2} - 2$		M1 A1	
	$\tan\psi = \frac{dy}{dx} = t$, so $(1+t^2)^{3/2} = \sec^3\psi$	Hence $s = 2(\sec^3\psi - 1)$	M1 A1	7
4.	(a) If $\frac{d^2y}{dx^2} = 9y$ then $k^2 = 9$ $k = 3$		M1 A1 A1	
	(b) $y(0) = 2 : a = 2$	$\frac{dy}{dx} = ka\cosh x + kb\sinh x$ $y'(0) = 1 : b = 1$	B1 M1 A1	
	$y = 2\cosh 3x + \frac{1}{3}\sinh 3x$		A1	
	(c) If $y = 0$, $\tanh 3x = -6$. This has no solution, since $-1 < \tanh 3x < 1$		M1 A1 A1	10
5.	(a) Curve sketched (odd function, rotational symmetry about origin)		B2	
	(b) Area $= \int_0^{1/2} \arcsin x\,dx$	Let $u = \arcsin x$, $dv = dx$	B1 M1	
	$du = \frac{1}{\sqrt{1-x^2}}\,dx$, $v = x$		A1 A1	
	$\int_0^{1/2} \arcsin x\,dx = x\arcsin x - \int_0^{1/2} x(1-x^2)^{-1/2}\,dx$		M1 A1 A1	
	$= [x\arcsin x + (1-x^2)^{1/2}]_0^{1/2} = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$		M1 A1	11
6.	(a) $\frac{d}{dx}(\ln x)^n = \frac{n(\ln x)^{n-1}}{x}$		M1 A1	
	(b) Let $u = (\ln x)^n$, $dv = x\,dx$	$du = \frac{n(\ln x)^{n-1}}{x}$, $v = \frac{x^2}{2}$	M1 A1	
	$I_n = \left[\frac{x^2}{2}(\ln x)^n\right]_1^e - \frac{n}{2}\int_1^e x(\ln x)^{n-1}dx = \frac{e^2}{2} - \frac{n}{2}I_{n-1}$		M1 A1 A1	
	(c) $I_0 = \frac{e^2-1}{2}$ $I_1 = \frac{e^2}{2} - \frac{1}{2}\left(\frac{e^2-1}{2}\right) = \frac{e^2+1}{4}$	$I_2 = \frac{e^2}{2} - I_1 = \frac{e^2-1}{4}$	B1 M1 A1 M1 A1	
				12

7. (a) $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} = \frac{6a\tan^2 t\sec^2 t}{6a\sec^2 t\tan t} = \tan t$ $\left(1+\left(\frac{dy}{dx}\right)^2\right)^{1/2} = \sec t$ M1 A1 A1

Area = $2\pi\int_{\pi/4}^{0}\sec t.6a\sec^2 t\tan t\ dt = 6a\int_{\pi/4}^{0}\sec^2 t\ d(\sec t)$ M1 A1

$= \left[2a\sec^3 t\right]_{\pi/4}^{0} = 2a(1-2\sqrt{2})$ M1 A1

(b) $\frac{dy}{dx} = \frac{d}{dt}(\tan t)\frac{dt}{dx} = \frac{\sec^2 t}{6a\sec^2 t\tan t} = \frac{1}{6a\tan t} = \frac{1}{6a}$ when $t = \frac{\pi}{4}$ M1 A1 A1

$\rho = \frac{(1+1)^{3/2}}{1/6a} = 12a\sqrt{2}$ M1 A1 12

8. (a) $\frac{dy}{dx} = \frac{dy}{dp} / \frac{dx}{dp} = \frac{-c}{p^2}.\frac{1}{c} = -\frac{1}{p^2}$ Gradient of normal $= p^2$ M1 A1 A1

Normal is $y - \frac{c}{p} = p^2(x-cp)$ $py - c = p^3x - cp^4$ M1 A1

(b) Passes through (0, 0) if $p = 1$ or $p = -1$ M1 A1

(c) At Q, $y = \frac{c(1-p^4)}{p}$ At mid-point, $x = \frac{cp}{2}$, so $p = \frac{2x}{c}$ B1 B1

$y = \frac{1}{2}\left(\frac{c}{p} + \frac{c(1-p^4)}{p}\right) = \frac{c(2-p^4)}{2p} = \frac{c^2}{4x}\left(2 - \frac{16x^4}{c^4}\right)$ M1 A1

$y = \frac{c^2}{2x} - \frac{4x^3}{c^2}$ A1 12

