PURE MATHS 5 (A) TEST PAPER 3 : ANSWERS AND MARK SCHEME

1. If $y = \arccos(2x)$, $\dfrac{dy}{dx} = \dfrac{-2}{\sqrt{1-4x^2}}$ When $x = \dfrac{1}{8}$, $\dfrac{dy}{dx} = \dfrac{-2}{\sqrt{1-1/16}} = \dfrac{-8}{\sqrt{15}}$ M1 A1 A1 3

2. If $y = mx + 2$ touches $x^2 + 2y^2 = 3$, then $x^2 + 2(mx+2)^2 = 3$ has one real root M1 A1
Equation is $(1 + 2m^2)x^2 + 8mx + 5 = 0$ M1 A1
Need $(8m)^2 - 4(5)(1 + 2m^2) = 0$ $24m^2 - 20 = 0$ $m = \pm\sqrt{\dfrac{5}{6}}$ M1 A1 6

3. (a) $x = \sinh y$ $\dfrac{dx}{dy} = \cosh y = \sqrt{1+\sinh^2 y}$ $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1+x^2}}$ M1 A1 A1
(b) $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}(1+x^2)^{-1/2} = -\dfrac{1}{2}(1+x^2)^{-3/2}.2x = \dfrac{-x}{(1+x^2)^{3/2}}$ M1 A1
$(x^2+1)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = \dfrac{-x}{(1+x^2)^{1/2}} + x.\dfrac{1}{(1+x^2)^{1/2}} = 0$ M1 A1 7

4. (a) $\dfrac{dy}{dx} = \tan x$ $s = \int_0^x \sqrt{1+\tan^2 u}\, du = \int_0^x \sec u\, du = \ln(\sec x + \tan x)$ B1 M1 A1 A1
(b) $\dfrac{dy}{dx} = \tan\psi$, so $\psi = x$. Hence $s = \ln(\sec\psi + \tan\psi)$ M1 A1 A1
(c) $\rho = \dfrac{ds}{d\psi} = \sec\psi = \sqrt{2}$ M1 A1 A1 10

5. (a) By substitution $x = 3\sinh u$, $I = \left[\operatorname{arsinh}\dfrac{x}{3}\right]_0^4 = \operatorname{arsinh}\dfrac{4}{3} = \ln 3$ M1 A1 A1
(b) Sketch of $y = 1/\sqrt{9+x^2}$ and region below curve between $x = 0$ and $x = 4$ B3
(c) Volume $= \pi\int_0^4 \dfrac{1}{9+x^2}\,dx = \pi\left[\dfrac{1}{3}\arctan\left(\dfrac{x}{3}\right)\right]_0^4 = \dfrac{\pi}{3}\arctan\dfrac{4}{3} \approx 0.971$ M1 A1 M1 A1
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6. (a) $\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{\frac{1}{2}(e^x + e^{-x})}{\frac{1}{2}(e^x - e^{-x})} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$ M1 A1
(b) Let $y = \operatorname{arcoth} x$, so $x = \coth y$ M1
Then $x = \dfrac{e^y + e^{-y}}{e^y - e^{-y}}$ $xe^y - xe^{-y} - e^y - e^{-y} = 0$ A1 A1
$(x-1)e^{2y} - (x+1) = 0$ $e^{2y} = \dfrac{x+1}{x-1}$ $y = \dfrac{1}{2}\ln\left(\dfrac{x+1}{x-1}\right)$ M1 A1
(c) $\ln\left((x-1)\sqrt{\dfrac{x+1}{x-1}}\right) = 3$ $\ln\sqrt{(x^2-1)} = 3$ M1 A1
$\ln(x^2 - 1) = 6$ $x^2 - 1 = e^6$ $x = 20.1$ M1 A1 11

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7. (a) Let $u = \cosh ax$, so $du = a \sinh ax\, dx$	B1	
$\int \frac{u^n}{a}\, du = \frac{u^{n+1}}{a(n+1)} + c = \frac{\cosh^{n+1} ax}{a(n+1)} + c$	M1 A1 A1	
(b) Let $u = \cosh^{n-1} ax$, $dv = \cosh ax\, dx$	M1	
$du = a(n-1)\cosh^{n-2} ax \sinh ax\, dx$, $v = \frac{1}{a}\sinh ax$	A1	
Integral = $\frac{1}{a}\sinh ax \cosh^{n-1} ax - (n-1)\int \cosh^{n-2} ax \sinh^2 ax\, dx$	M1 A1	
= $\frac{1}{a}\sinh ax \cosh^{n-1} ax - (n-1)\int \cosh^n ax - \cosh^{n-1} ax\, dx$	M1 A1	
= $\frac{1}{a}\sinh ax \cosh^{n-1} ax - (n-1)(I_n - I_{n-2})$	A1	
Hence $nI_n = \frac{1}{a}\sinh ax \cosh^{n-1} ax + (n-1)I_{n-2}$	A1	12
8. (a) $xy = c^2$ Curve sketched, with axes as asymptotes	B1 B2	
(b) Tangent at P has gradient $\frac{dy}{dx} = \frac{-1}{p^2}$	M1 A1	
Equation of tangent is $y - \frac{c}{p} = -\frac{1}{p^2}(x - cp)$ $p^2y + x = 2cp$	M1 A1	
Similarly, tangent at Q is $q^2y + x = 2cq$ Where these meet,	B1	
$p^2y - 2cp = q^2y - 2cq$ $(p^2 - q^2)y = 2c(q - p)$	M1	
$y = \frac{2c}{p+q}$ Then $x = 2cp - p^2\frac{2c}{p+q} = \frac{2cpq}{p+q}$	A1 A1	
(c) When $q = 2p$, $x = \frac{2cp.2p}{3p} = \frac{4cp}{3}$ $y = \frac{2c}{3p}$	M1 A1 A1	
$xy = \frac{8c^2}{9}$, which is a rectangular hyperbola	A1 A1	16