PURE MATHS 5 (A) TEST PAPER 2 : ANSWERS AND MARK SCHEME

1. $x = \tan y \qquad \frac{dx}{dy} = \sec^2 y = 1 + \tan^2 y = 1 + x^2 \qquad \frac{dy}{dx} = \frac{1}{1+x^2}$ M1 A1 A1 3

2. (a) $7(e^x - e^{-x}) = 48 \qquad 7e^{2x} - 48e^x - 7 = 0 \qquad (7e^x + 1)(e^x - 7) = 0$ M1 A1
$e^x = 7 \qquad x = \ln 7$ A1
(b) $\cosh x = \frac{1}{2}(7 + \frac{1}{7}) = \frac{25}{7} \qquad \sinh 2x = 2\sinh x \cosh x = \frac{1200}{49}$ B1 M1 A1 6

3. (a) Let $u = \sin^{n-1} x$, $dv = \sin x\, dx \qquad du = (n-1)\sin^{n-2} x \cos x\, dx,\ v = -\cos x$ M1 A1
$I_n = [-\sin^{n-1} x \cos x]_0^{\pi} + \int_0^{\pi} (n-1)\sin^{n-2} x \cos^2 x\, dx$ M1 A1
$= (n-1)\int_0^{\pi} \sin^{n-2} x - \sin^n x\, dx = (n-1)I_{n-2} - (n-1)I_n \qquad nI_n = (n-1)I_{n-2}$ M1 A1
(b) $I_1 = [\cos x]_0^{\pi} = 1 - (-1) = 2 \qquad I_3 = \frac{2}{3}(2) = \frac{4}{3} \qquad I_3 = \frac{4}{5}\left(\frac{4}{3}\right) = \frac{16}{15}$ M1 A1 A1 A1
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4. (a) Parabola B1
(b) $\frac{dy}{dx} = \frac{6}{6t} = \frac{1}{t} \qquad \frac{d^2y}{dx^2} = -\frac{1}{t^2}.\frac{1}{6t} = -\frac{1}{6t^3}$ M1 A1
When $t = -1$, $\frac{dy}{dx} = -1$ and $\frac{d^2y}{dx^2} = \frac{1}{6} \qquad \rho = 6(2^{3/2}) = 12\sqrt{2}$ M1 A1
(c) Tangent has equation $y - 6p = \frac{1}{p}(x - 3p^2)$ M1 A1
If $y = 1$ when $x = 0$, $1 - 6p = -3p \qquad p = \frac{1}{3}$ M1 A1 9

5. (a) (i) $\int \frac{1}{\sqrt{(x+4)^2 + 4}}\, dx = \text{arsinh}\left(\frac{x+4}{2}\right) + c$ M1 A1 A1
(ii) $\int \frac{1}{(x+4)^2 + 4}\, dx = \frac{1}{2}\arctan\left(\frac{x+4}{2}\right) + c$ M1 A1 A1
(b) $\int_{-6}^{-2} \frac{4}{x^2 + 8x + 20}\, dx = \left[2\arctan\left(\frac{x+4}{2}\right)\right]_{-6}^{-2} = 2\frac{\pi}{4} - 2\left(-\frac{\pi}{4}\right) = \pi$ M1 A1 A1 9

6. (a) $\frac{dy}{dx} = x^{1/2} \qquad$ Arc length $= \int_0^3 (1 + (x^{1/2})^2)^{1/2}\, dx$ B1 M1 A1
$= \int_0^3 (1+x)^{1/2}\, dx$ A1
(b) Let $1 + x = u^2$, so $dx = 2u\, du \qquad$ Limits : $u = 1$, $u = 2$ B1 B1
Integral becomes $\int_1^2 2u^2\, du = \left[\frac{2u^3}{3}\right]_1^2 = \frac{14}{3}$ M1 A1 A1 9

7.	(a) Curve sketched, symmetrical about y-axis	B2	
	(b) When $x = \ln 2$, $y = \cosh(\ln 2) = \frac{5}{4}$	M1 A1	
	$\frac{dy}{dx} = \sinh x = \sinh(\ln 2) = \frac{3}{4}$, so gradient of normal $= -\frac{4}{3}$	M1 A1	
	Normal is $y - \frac{5}{4} = -\frac{4}{3}(x - \ln 2)$	M1	
	When $y = 0$, $x - \ln 2 = \frac{15}{16}$ $\quad x = \ln 2 + \frac{15}{16}$	A1 A1	
	(c) Vol. $= \pi\int_0^{\ln 2}\cosh^2 x\,dx + \frac{1}{3}\pi\left(\frac{5}{4}\right)^2\frac{15}{16} = \frac{\pi}{2}\int_0^{\ln 2}(\cosh 2x + 1)dx + \frac{125\pi}{256}$	M1 A1 A1	
	$= \frac{\pi}{2}\left[\sinh 2x + 2x\right]_0^{\ln 2} + \frac{125\pi}{256} = \pi\left[\frac{15}{8} + 2\ln 2 + \frac{125}{256}\right] = \frac{245\pi}{256} + \frac{\pi}{2}\ln 2$	M1 A1	14
8.	(a) $\frac{dy}{d\theta} = \frac{5}{4}\cos\theta$ $\quad \frac{dx}{d\theta} = -\frac{5}{3}\sin\theta$ $\quad \frac{dy}{dx} = -\frac{3\cos\theta}{4\sin\theta}$	M1 A1	
	Tangent is $y - \frac{5}{4}\sin\theta = -\frac{3\cos\theta}{4\sin\theta}\left(x - \frac{5}{3}\cos\theta\right)$	M1 A1	
	$(4\sin\theta)y - 5\sin^2\theta = (-3\cos\theta)x + 5\cos^2\theta$	A1	
	$(4\sin\theta)y + (3\cos\theta)x = 5(\cos^2\theta + \sin^2\theta) = 5$	A1	
	(b) At P, $x = \frac{5}{3\cos\theta}$ $\quad$ At Q, $y = \frac{5}{4\sin\theta}$	B1 B1	
	Area $= \frac{1}{2}\,\frac{5}{3\cos\theta}\,\frac{5}{4\sin\theta} = \frac{25}{12}.\frac{1}{2\sin\theta\cos\theta} = \frac{25}{12}\operatorname{cosec} 2\theta$	M1 A1	
	(c) Mid-point of PQ is $\left(\frac{5}{6\cos\theta}, \frac{5}{8\sin\theta}\right)$	B1	
	$\cos\theta = \frac{5}{6x}$, $\sin\theta = \frac{5}{8y}$ $\quad$ Hence $\frac{25}{36x^2} + \frac{25}{64y^2} = 1$	M1 A1	
	$\frac{25(16y^2 + 9x^2)}{576x^2y^2} = 1$ $\quad 9x^2 + 16y^2 = \frac{576}{25}x^2y^2$	M1 A1	15