PURE MATHS 5 (A) TEST PAPER 10 : ANSWERS AND MARK SCHEME

1. $\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}} = \frac{4}{\sqrt{3}}$ at given point $\quad y = \frac{\pi}{6}$ M1 A1 B1

$y - \frac{\pi}{6} = -\frac{\sqrt{3}}{4}\left(x - \frac{1}{4}\right) \quad 48y + 12\sqrt{3}\,x = 8\pi + 3\sqrt{3}$ M1 A1 5

2. $\int_0^3 \frac{1}{2}\sinh 2x\,dx = \left[\frac{1}{4}\cosh 2x\right]_0^3 = \frac{1}{4}(\cosh 6 - \cosh 0) = \frac{1}{4}\left(\frac{e^6 + e^{-6}}{2} - 1\right)$ B1 M1 A1 M1 A1

5

3. (a) $\coth^2 x - \operatorname{cosech}^2 x = \left(\frac{e^x + e^{-x}}{e^x - e^{-x}}\right)^2 - \left(\frac{1}{e^x - e^{-x}}\right)^2 = \frac{e^{2x} + e^{-2x} + 2 - 4}{(e^x - e^{-x})^2}$ M1 A1

$= \frac{e^{2x} + e^{-2x} - 2}{e^{2x} + e^{-2x} - 2} = 1$ A1

(b) $1 + \operatorname{cosech}^2 x = 2\operatorname{cosech} x \quad \operatorname{cosech}^2 x - 2\operatorname{cosech} x + 1 = 0$ M1 A1

$(\operatorname{cosech} x - 1)^2 = 0 \quad \operatorname{cosech} x = 1 \quad \sinh x = 1$ M1 A1

$e^x - e^{-x} = 2 \quad e^{2x} - 2e^x - 1 = 0 \quad e^x = (2 \pm \sqrt{8})/2 = 1 \pm \sqrt{2}$ M1 A1 A1

$x = \ln(1 + \sqrt{2})$ A1 11

4. (a) $P = (ap^2, 2ap), \quad Q = (a(p+k)^2, 2a(p+k))$ B1 B1

Gradient of $PQ = \frac{2ak}{a(k^2 + 2pk)} = \frac{2}{k + 2p}$ M1 A1

Equation of PQ is $y - 2ap = \frac{2}{k+2p}(x - ap^2)$ M1 A1

(b) When $k = 0$, equation becomes $y - 2ap = \frac{1}{p}(x - ap^2) \quad py = x + ap^2$ M1 A1

(c) $\frac{1}{p}\frac{1}{k+p} = -1 \quad 1 = -p(k+p) \quad p^2 + kp + 1 = 0$ M1 A1 A1 11

5. (a) Curve sketched, symmetric about y-axis B2

(b) $f'(x) = \frac{1}{3}[\sinh x\,(2\cosh x \sinh x) + \cosh x(2 + \cosh^2 x)]$ M1 A1

$= \frac{2}{3}\cosh x \sinh^2 x + \frac{1}{3}\cosh^3 x + \frac{2}{3}\cosh x$ A1

$= \frac{2}{3}\cosh x(\cosh^2 x - 1) + \frac{1}{3}\cosh^3 x + \frac{2}{3}\cosh x = \cosh^3 x$ M1 A1

(c) Volume $= \pi\int_0^{\ln 3}\cosh^3 x\,dx = \frac{\pi}{3}\left[\sinh x(2 + \cosh^2 x)\right]_0^{\ln 3} = \frac{\pi}{3}\left[\frac{4}{3}\left(2 + \frac{25}{9}\right)\right]$ M1 A1 M1 A1

$= \frac{172\pi}{81}$ A1 12

6.	(a) $y = \int\sqrt{9+x^2}\,dx$	Let $x = 3\sinh t$, so $dx = 3\cosh t\,dt$	B1 M1 A1	
	$y = 9\int\cosh^2 t\,dt = \frac{9}{2}\int\cosh 2t + 1\,dt = \frac{9}{2}\left[\frac{1}{2}\sinh 2t + t\right]$		M1 A1 A1	
	$y = 2\frac{x}{3}\frac{\sqrt{9+x^2}}{3} + \frac{9}{2}\text{arsinh}\,\frac{x}{3} + c$	$(0, 1) : c = 1$	M1 A1 A1	
	$y = \frac{1}{2}x\sqrt{9+x^2} + \frac{9}{2}\text{arsinh}\,\frac{x}{3} + 1$		A1	
	(b) $\frac{dy}{dx} = \sqrt{9+x^2} = \sqrt{10}$ when $x = 1$	$\frac{d^2y}{dx^2} = \frac{1}{2}(9+x^2)^{-1/2}.2x = \frac{1}{\sqrt{10}}$	B1 M1 A1	
	$\rho = 11\sqrt{110}$		M1 A1	15
7.	(a) Let $u = x^n$, $dv = \cosh x\,dx$	$du = nx^{n-1}\,dx$, $v = \sinh x$	M1 A1	
	$I_n = [x^n \sinh x]_0^1 - n\int_0^1 x^{n-1}\sinh x\,dx$		M1 A1	
	Let $u = x^{n-1}$, $dv = \sinh x\,dx$	$du = (n-1)x^{n-2}\,dx$, $v = \cosh x$	M1 A1	
	$I_n = [x^n\sinh x - nx^{n-1}\cosh x]_0^1 - n(n-1)\int_0^1 x^{n-2}\cosh x\,dx$		M1 A1	
	$I_n = \sinh 1 - n\cosh 1 + n(n-1)I_{n-2}$		M1 A1 A1	
	(b) $I_0 = \int_0^1 \cosh x\,dx = \sinh 1$		B1	
	$I_2 = \sinh 1 - 2\cosh 1 + 2\sinh 1 = 3\sinh 1 - 2\cosh 1$		M1 A1	
	$I_4 = \sinh 1 - 4\cosh 1 + 12(3\sinh 1 - 2\cosh 1) = 37\sinh 1 - 28\cosh 1$		M1 A1	16