1. Find the coordinates of any point(s) of intersection of the curves with equations $y = \cosh 2x$ and $y = 3 - 2\cosh x$. **(5 marks)**

2. (a) Prove that $\frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}$. **(3 marks)**

   (b) Find the gradient of the curve $y = \arccos(\ln 2x)$ at the point where $x = \frac{1}{2}$. **(3 marks)**

3. The parametric equations of a curve C are

$$x = 3t^2, \quad y = 2t^3, \quad \text{where } t \geq 0.$$

   Prove that, with the usual notation, C has intrinsic equation $s = 2(\sec^3 \psi - 1)$. **(7 marks)**

4. (a) Find the positive value of k for which $y = a\cosh kx + b\sinh kx$ is a solution of the differential equation

$$\frac{d^2y}{dx^2} - 9y = 0.$$

   (3 marks)

   (b) Hence find a solution of this equation for which $y = 2$ and $\frac{dy}{dx} = 1$ when $x = 0$. **(4 marks)**

   (c) Show that the graph of this solution does not cross the x-axis. **(3 marks)**

5. (a) Sketch the curve with equation $y = \arcsin x$ for $-1 \leq x \leq 1$. **(2 marks)**

   (b) Find the area of the region bounded by the curve $y = \arcsin x$, the x-axis and the line $x = \frac{1}{2}$. **(9 marks)**

6. (a) Find $\frac{d}{dx}(\ln x)^n$, where $n \geq 0$. **(2 marks)**

   Given that $I_n = \int_1^e x(\ln x)^n \, dx$,

   (b) show that $2I_n = e^2 - nI_{n-1}$. **(5 marks)**

   (c) Hence find the exact value of I_2. **(5 marks)**

7. The parametric equations of a curve are

$$x = 3a \sec^2 t, \; y = 2a \tan^3 t,$$

where $a > 0$ and $0 \leq t \leq \frac{\pi}{4}$.

(a) Find the area of the surface formed when the curve is rotated once about the x-axis.

(7 marks)

(b) Find the radius of curvature at the point where $t = \frac{\pi}{4}$. **(5 marks)**

8. The point P lies on the rectangular hyperbola with equation $xy = c^2$, where $c \neq 0$.

The x-coordinate of P is cp.

(a) Show that the normal to the hyperbola at P has equation $py - c = p^3(x - cp)$. **(5 marks)**

(b) Find the values of p for which this normal passes through the origin. **(2 marks)**

This normal meets the y-axis at the point Q.

(c) Find an equation of the locus of the mid-point of PQ, as P moves on the hyperbola.

(5 marks)