PURE MATHS 5 (A) TEST PAPER 6 : ANSWERS AND MARK SCHEME

1.	$7 = \cosh 2t_1$	$2\cosh^2 t_1 - 1 = 7$	$\cosh t_1 = 2$	M1 A1	
	$e^{t_1} + e^{-t_1} = 4$	$e^{2t_1} - 4e^{t_1} + 1 = 0$	$e^{t_1} = \dfrac{4 \pm \sqrt{12}}{2}$	M1 A1	
	$e^{t_1} = 2 \pm \sqrt{3}$	$t_1 = \ln(2 + \sqrt{3})$		M1 A1	6

2. (a) Sketch : s = arc length from P to a general point, ψ = angle between tangent at P and x-axis — B2

(b) $\dfrac{ds}{d\psi} = s + 1$, so $\psi = \ln(s+1) + c$ $\quad s + 1 = Ae^{\psi}$ — M1 A1 A1

$s = 0$ when $\psi = 0$, so $A = 1$ $\quad s = e^{\psi} - 1$ — M1 A1 — 7

3. (a) $\int \dfrac{1}{u^2+9}\,du = \dfrac{1}{3}\arctan\left(\dfrac{u}{3}\right) + c = \dfrac{1}{3}\arctan\left(\dfrac{x-2}{3}\right) + c$ — M1 A1 A1

(b) $\int \dfrac{1}{\sqrt{16-u^2}}\,du = \arcsin\left(\dfrac{u}{4}\right) + c = \arcsin\left(\dfrac{x-2}{4}\right) + c$ — B1 M1 A1 A1 — 7

4. (a) $a(4\sec^2\theta) - b(9\tan^2\theta) = 1$ $\quad a = \dfrac{1}{4},\ b = \dfrac{1}{9}$ — M1 A1 A1

(b) $\dfrac{dy}{dx} = \dfrac{3\sec^2\theta}{2\sec\theta\tan\theta} = \dfrac{3\sec\theta}{2\tan\theta}$ — M1 A1

Normal is $y - 3\tan\theta = -\dfrac{2\tan\theta}{3\sec\theta}(x - 2\sec\theta)$ — M1

$(3\sec\theta)y + (2\tan\theta)x = 13\sec\theta\tan\theta$ — A1

(c) $(0, 1)$: $3\sec\theta = 13\sec\theta\tan\theta$ $\quad \tan\theta = 3/13$ $\quad \theta = 0.23, 6.51$ — M1 A1

$(1, 0)$: $\sec\theta = 2/13$, so $\cos\theta = 13/2$, > 1, so no values of θ — M1 A1 — 11

5. (a) Let $u = x\sinh^{n-1}x$, $dv = \sinh x\,dx$ — M1

$du = \sinh^{n-1}x + (n-1)x\sinh^{n-2}x\cosh x\,dx$, $\quad v = \cosh x$ — A1 A1

$I_n = x\sinh^{n-1}x\cosh x - \int \sinh^{n-1}x\cosh x\,dx - (n-1)\int x\sinh^{n-2}x\cosh^2 x\,dx$ — M1 A1

$= x\sinh^{n-1}x\cosh x - \frac{1}{n}\sinh^n x - (n-1)\int (x\sinh^{n-2}x + x\sinh^n x)\,dx$ — M1 A1

Hence $[1 + (n-1)]I_n = nI_n = x\sinh^{n-1}x\cosh x - \frac{1}{n}\sinh^n x - (n-1)I_{n-2}$ — M1 A1

(b) $I_0 = \int x\,dx = \frac{1}{2}x^2$ $\quad 2I_2 = x\sinh x\cosh x - \frac{1}{2}\sinh^2 x - \frac{1}{2}x^2$ — B1 M1 A1

$[I_2]_0^{\ln 2} = \dfrac{15}{32}\ln 2 - \dfrac{1}{4}(\ln 2)^2 - \dfrac{9}{64}$ — M1 A1 — 14

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6. (a) $\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$, $\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$, so $\frac{dy}{dx} = -\tan\theta$ M1 A1 A1

$\left(1+\left(\frac{dy}{dx}\right)^2\right)^{1/2} = \sec\theta$ Arc = $\int_{\pi/3}^{0}\sec\theta(-3a\cos^2\theta\sin\theta\, d\theta)$ M1 A1 M1

$= -\frac{3a}{2}\int_{\pi/3}^{0}\sin 2\theta\, d\theta = \frac{3a}{2}\left[\frac{\cos 2\theta}{2}\right]_{\pi/3}^{0} = \frac{3a}{4}\left[1+\frac{1}{2}\right] = \frac{9a}{8}$ A1 M1 A1

(b) Area = $2\pi\int_{\pi/3}^{0} a\sin^3\theta\sec\theta(-3a\cos^2\theta\sin\theta\, d\theta)$ M1 A1

$= -6\pi a^2\int_{\pi/3}^{0}\sin^4\theta\cos\theta\, d\theta = -6\pi a^2\left[\frac{\sin^5\theta}{5}\right]_{\pi/3}^{0} = \frac{27\sqrt{3}\pi a^2}{80}$ M1 A1 A1 14

7. (a) Integrating factor = $e^{\cosh x}$ $e^{\cosh x}\frac{dy}{dx} + (\sinh x\, e^{\cosh x})y = x$ B1 M1 A1

$\frac{d}{dx}(e^{\cosh x}y) = x$ $e^{\cosh x}y = \frac{1}{2}x^2 + c$ $y = e^{-\cosh x}(\frac{1}{2}x^2 + c)$ A1 M1 A1

(b) Auxiliary equation $u^2 - 5u + 6 = 0$ has roots $u = 2$, $u = 3$ M1 A1
Complementary function : $y = Ae^{2x} + Be^{3x}$ A1
Let particular integral be $y = a\cosh 4x + b\sinh 4x$, so

$\frac{dy}{dx} = 4a\sinh 4x + 4b\cosh 4x$, $\frac{d^2y}{dx^2} = 16a\cosh 4x + 16b\sinh 4x$ B1 B1

Substituting in equation gives
$(22a - 20b)\cosh 2x + (22b - 20a)\sinh 2x = \cosh 2x - \sinh 2x$ M1 A1

$22a - 20b = 1$, $22b - 20a = -1$ $a = \frac{1}{42}$, $b = -\frac{1}{42}$ M1 A1

General solution is $y = Ae^{2x} + Be^{3x} + \frac{1}{42}(\cosh 4x - \sinh 4x)$ A1 16

