1. Given that $2\frac{dy}{dx} = 1 + y^2$, and that $y = 1$ when $x = 0$, find y in terms of x. **(6 marks)**

2. (a) Differentiate $\arccos(2x)$ with respect to x. **(3 marks)**

   (b) Evaluate $\int_0^{1/4} \frac{3}{\sqrt{(1-4x^2)}}\, dx$, giving your answer to 3 significant figures. **(3 marks)**

3. Starting from the definition of cosh in terms of exponential functions, prove that

$$\text{arcosh}\, x = \ln[x + \sqrt{(x^2 - 1)}].$$

   Hence find the exact value of $\text{arcosh}\,\frac{13}{12}$, in terms of natural logarithms. **(7 marks)**

4. The parabola with equation $y^2 = 4ax$ passes through the point P with coordinates (6, 6)

   (a) Find a and write down parametric equations for the parabola. **(3 marks)**

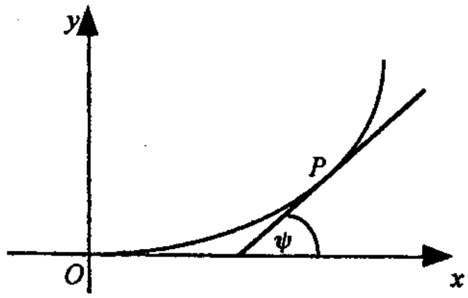
   (b) Find the radius of curvature of the parabola at the point $(\frac{3}{2}, 3)$. **(5 marks)**

5. If $I_n = \int x^n(1 + x^2)^5\, dx$, prove that for $n \geq 2$,

$$f(n)I_n = x^{n-1}(1 + x^2)^6 - (n - 1)I_{n-2},$$

   where $f(n)$ is a linear function of n to be found. **(10 marks)**

6. The diagram shows the curve whose equation is $y = \frac{x^2}{2}$, $x \geq 0$.
   The angle between the tangent at $P(x, y)$ and the x-axis is ψ. The arc length from O to P is s.



   (a) Show that $s = \int_0^{\text{arsinh}\, x} \cosh^2 u\, du$ and that $x = \tan\psi$. **(7 marks)**

   (b) Deduce that the intrinsic equation of the curve is $s = \frac{1}{2}[\sec\psi\tan\psi + \text{arsinh}(\tan\psi)]$. **(5 marks)**

7. The arc l joins the points $(1, 2)$ and $(8, 4\sqrt{2})$ on the curve with equation $y = 2\sqrt{x}$.

   (a) Show that the length of l is given by

   $$\int_1^8 \sqrt{1+\frac{1}{x}}\ \mathrm{d}x$$

   and use the trapezium rule, with seven strips of equal width, to estimate this integral. Give your answer to 2 significant figures. **(6 marks)**

   (b) Show that the area of the curved surface formed when l is rotated once about the x-axis is

   $$\frac{8\pi}{3}(27 - 2\sqrt{2}).$$ **(5 marks)**

8. The curve C is the ellipse with parametric equations $x = a\cos\theta,\ y = ka\sin\theta$, where k and a are real constants and $k < 1$.

   (a) Find a cartesian equation of C. **(2 marks)**

   (b) State (i) the eccentricity of C, (ii) the coordinates of the foci of C. **(4 marks)**

   (c) Show that if the line $y = mx + c$ is a tangent to C, then $a^2(m^2 + k^2) = c^2$. **(5 marks)**

   (d) Deduce the values of m for which the line $y = mx + 9$ is a tangent to the ellipse $x^2 + 4y^2 = 9$. **(4 marks)**