1. Find the exact value of $\int_0^1 \frac{1}{\sqrt{x^2+8x}}\,dx$. **(6 marks)**

2. Find the general solution of the differential equation

$$\cosh x \frac{dy}{dx} + y \sinh x = 1.$$ **(7 marks)**

3. Given that $I_n = \int_0^1 x^n e^{2x}\,dx$, where $n \geq 0$,

   (a) show that, for $n \geq 1$, $2I_n = e^2 - nI_{n-1}$. **(5 marks)**

   (b) Find the exact value of I_0. **(2 marks)**

   (c) Hence express I_2 in its simplest form in terms of e. **(3 marks)**

4. (a) Given that $y = \text{arcosh } 2x$, prove that $\frac{dy}{dx} = \frac{2}{\sqrt{4x^2-1}}$. **(4 marks)**

   (b) Find $\int \text{arcosh } 2x\,dx$. **(7 marks)**

5. (a) Using the substitution $u = e^x$, or otherwise, find $\int \text{sech } x\,dx$. **(7 marks)**

   The region R is bounded by the curve with equation $y = \text{sech } x$, the x-axis and the lines $x = 1$ and $x = \ln 5$.

   (b) Draw a sketch to show the region R. **(2 marks)**

   (c) Calculate the area of R, correct to 3 significant figures. **(3 marks)**

6. The parametric equations of a curve are

$$x = a(1 - \cos 2t), \quad y = a(2t + \sin 2t),$$

   where a is a non-zero real constant and $0 \leq t \leq \frac{\pi}{2}$.

   (a) Show that the length of the curve is $4a$. **(9 marks)**

   (b) Find the radius of curvature of the curve at the point where $t = \frac{\pi}{4}$. **(5 marks)**

7. The point $P\,(a \cosh p, b \sinh p)$, where $a \neq 0$ and $b \neq 0$, lies on a hyperbola.

   (a) Write down the cartesian equation of this hyperbola. **(2 marks)**

   (b) State the equations of the asymptotes of the hyperbola. **(2 marks)**

   (c) Find an equation of the tangent to the hyperbola at P. **(5 marks)**

   The tangent at P meets the asymptotes of the hyperbola at A and B.

   (d) Show that P is the mid-point of AB. **(6 marks)**