1. Find the gradient of the curve with equation $y = \arccos(2x)$ at the point where $x = \frac{1}{8}$. **(3 marks)**

2. Find, in surd form, the values of m for which the straight line $y = mx + 2$ is a tangent to the ellipse with equation $x^2 + 2y^2 = 3$. **(6 marks)**

3. Given that $y = \text{arsinh}\, x$,
   (a) find $\frac{dy}{dx}$ in terms of x. **(3 marks)**
   (b) Find the value of $(x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$. **(4 marks)**

4. The cartesian equation of a curve C is $y = \ln(\sec x)$, $0 \le x < \frac{\pi}{2}$.
   (a) Find, in terms of x, the length of the arc of C from O (0, 0) to P (x, y). **(4 marks)**
   (b) By considering the gradient of the tangent at P, find the intrinsic equation of C in the form $s = f(\psi)$. **(3 marks)**
   (c) Find the radius of curvature of C at the point where $x = \frac{\pi}{4}$. **(3 marks)**

5. Given that $I = \int_0^4 \frac{1}{\sqrt{9 + x^2}}\, dx$,
   (a) evaluate I. **(3 marks)**
   (b) Sketch a diagram to show a region R whose area is given by I. **(3 marks)**
   (c) Calculate the volume formed when R is rotated through 360° about the x-axis. **(4 marks)**

6. (a) Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, show that
   $$\coth x \equiv \frac{e^x + e^{-x}}{e^x - e^{-x}}.$$ **(2 marks)**
   (b) Given that $x > 1$, deduce find an expression for $\text{arcoth}\, x$ in logarithmic form. **(5 marks)**
   (c) Solve the equation $\ln(x - 1) + \text{arcoth}\, x = 3$, giving the solution to 3 significant figures. **(4 marks)**

7. Given that a is a real constant,

   (a) find $\int \cosh^n ax \sinh ax \, dx$. **(4 marks)**

   (b) If $I_n = \int \cosh^n ax \, dx$, show that $nI_n = \frac{1}{a} \sinh ax \cosh^{n-1} ax + (n-1)I_{n-2}$. **(8 marks)**

8. The points $P\,(cp, \frac{c}{p})$ and $Q\,(cq, \frac{c}{q})$, where $p \neq 0$, $q \neq 0$, $q \neq p$, lie on a rectangular hyperbola.

   (a) State the equation of this hyperbola, and sketch the curve. **(3 marks)**

   The tangents to the hyperbola at P and Q meet at T.

   (b) Show that T has coordinates $\left(\frac{2cpq}{p+q}, \frac{2c}{p+q}\right)$. **(8 marks)**

   (c) If P and Q vary such that $p = 2q$, show that the locus of T is another rectangular hyperbola and give its equation. **(5 marks)**