1. Find, in the form $ax + by = c$, an equation of the normal to the curve $y = \arcsin(2x)$ at the point where $x = \frac{1}{4}$. **(5 marks)**

2. Find, in terms of e, the exact value of $\int_0^3 \sinh x \cosh x \, dx$. **(5 marks)**

3. (a) Starting from the definitions of the hyperbolic functions in terms of exponentials, prove that
$$\coth^2 x - \operatorname{cosech}^2 x = 1.$$ **(3 marks)**

   (b) Solve the equation $\coth^2 x = 2 \operatorname{cosech} x$, giving your answers in terms of natural logarithms. **(8 marks)**

4. A parabola has parametric equations $x = at^2$, $y = 2at$, where a is a non-zero constant.
   P and Q are the points on the parabola where $t = p$ and $t = p + k$, respectively.

   (a) Find an equation of the chord PQ. **(6 marks)**

   (b) Using your answer to (a), deduce the equation of the tangent to the parabola at P. **(2 marks)**

   (c) Show that if the tangents at P and Q are perpendicular, then $p^2 + kp + 1 = 0$. **(3 marks)**

5. (a) Sketch the graph of $y = \cosh x$. **(2 marks)**

   (b) If $f(x) = \frac{1}{3}\sinh x\,(2 + \cosh^2 x)$, show that $f'(x) = \cosh^3 x$. **(5 marks)**

   (c) The arc of the curve $y = (\cosh x)^{3/2}$ between the points where $x = 0$ and $x = \ln 3$ is rotated once completely about the x-axis. Calculate the volume of the solid generated. **(5 marks)**

6. The gradient G of a curve at the point (x, y) is given by $G = \sqrt{(9 + x^2)}$.
   The curve passes through the point $P\,(0, 1)$.

   (a) Find an equation of the curve in the form $y = f(x)$. **(10 marks)**

   (b) Find the radius of curvature of the curve at the point where $x = 1$. **(5 marks)**

7. (a) Given that $I_n = \int_0^1 x^n \cosh x \, dx$, where n is a positive integer, prove that, for $n \geq 2$,
$$I_n = \sinh 1 - n \cosh 1 + n(n-1)I_{n-2}.$$ **(11 marks)**

   (b) Hence find the value of I_4, leaving your answer in terms of hyperbolic functions. **(5 marks)**