1. The intrinsic equation of a curve, with the usual notation, is $s = \frac{1}{4}\tan\psi$, where $0 \le \psi \le \frac{\pi}{2}$. Find the values of ψ and s at the point on the curve where the radius of curvature is 1.

(5 marks)

2. Given that $f(x) = \arcsin\left(\frac{x}{2}\right)$, where $x \in \mathbb{R}, -2 < x < 2$,

   (a) define the inverse function f^{-1}, stating its domain. **(3 marks)**

   (b) Find $f'(x)$ in terms of x. **(3 marks)**

3. The part of the curve $y = \cosh x$ between $x = 0$ and $x = \ln 3$ is rotated through 360° about the x-axis. Show that the area of the curved surface formed is given by

$$2\pi \int_0^{\ln 3} \cosh^2 x \, dx$$

and find the exact value of this area. **(7 marks)**

4. (a) Starting from the definitions of $\cosh x$ and $\sinh x$ in terms of e^x, prove that

$$\cosh^2 x - \sinh^2 x \equiv 1.$$

(4 marks)

   (b) Find the exact values of $\sinh x$ for which $4\sinh^2 x = \cosh^2 x$, giving your answers in terms of natural logarithms. **(3 marks)**

5. (a) Find $\int \frac{1}{\sqrt{x^2 + 6x + 13}} \, dx$. **(4 marks)**

   (b) Hence find the area of the region bounded by the curve with equation $y = \frac{1}{\sqrt{x^2 + 6x + 13}}$, the x-axis and the lines $x = -3$ and $x = 3$. **(3 marks)**

6. (a) Given that $I_n = \int x^2 (\ln x)^n \, dx$, where $n \in \mathbb{N}$, show that, for $n \ge 1$, $3I_n = x^3 (\ln x)^n - nI_{n-1}$.

(6 marks)

   (b) Hence show that $\int_1^{e^2} x^2 (\ln x)^2 \, dx = \frac{1}{27}(26e^6 - 2)$. **(8 marks)**

7. The normals at $P\ (ap^2, 2ap)$ and $Q\ (aq^2, 2aq)$ to the parabola $y^2 = 4ax$ meet at N. $p \neq q$.

   (a) Show that the normal at P to the parabola has equation $y - 2ap = ap^3 - px$ and write down the equation of the normal at Q. **(5 marks)**

   (b) Show that the x-coordinate of N is $a(p^2 + q^2 + pq + 2)$ and find the y-coordinate of N. **(6 marks)**

   (c) If $p = 1$, show that as Q gets closer to P, N approaches the point $(5a, -2a)$. **(3 marks)**

8. The parametric equations of a curve C are $x = 1 + \sinh t$, $y = 5 - 4\cosh t$.

   (a) Show that C meets the x-axis at two points, and state their coordinates. **(7 marks)**

   (b) Sketch the curve C for $-1 \leq t \leq 1$. **(2 marks)**

   (c) Calculate the radius of curvature of C at the point where $t = \ln 2$. **(6 marks)**