PURE MATHS 5 (A) TEST PAPER 1 : ANSWERS AND MARK SCHEME

1. $\rho = \frac{ds}{d\psi} = \frac{1}{4}\sec^2\psi = 1$ when $\cos\psi = \frac{1}{2}$ M1 A1

$\psi = \frac{\pi}{3}$, $s = \frac{\sqrt{3}}{4}$ A1 M1 A1 5

2. (a) Let $y = f(x)$, so $x = 2\sin y$ $f^{-1}(y) = 2\sin y$, $-\frac{\pi}{2} < y < \frac{\pi}{2}$ B1 B1 B1

(b) $\frac{dx}{dy} = 2\cos y = \sqrt{(4 - x^2)}$ $f'(x) = \frac{dy}{dx} = \frac{1}{\sqrt{4-x^2}}$ M1 A1 A1 6

3. Area $= 2\pi\int_0^{\ln 3} y\sqrt{1+(y')^2}\,dx = 2\pi\int_0^{\ln 3}\cosh x\sqrt{1+\sinh^2 x}\,dx$ M1 A1

$= 2\pi\int_0^{\ln 3}\cosh^2 x\,dx = \pi\int_0^{\ln 3}(\cosh 2x + 1)\,dx = \pi\left[\frac{1}{2}\sinh 2x + x\right]_0^{\ln 3}$ A1 M1 A1

$= \pi\left[\frac{9-1/9}{4} + \ln 3\right] = \frac{20\pi}{9} + \pi\ln 3$ M1 A1 7

4. (a) $\cosh^2 x - \sinh^2 x = \frac{(e^x + e^{-x})^2}{4} - \frac{(e^x - e^{-x})^2}{4}$ M1 A1

$= \frac{(e^{2x} + e^{-2x} + 2) - (e^{2x} + e^{-2x} - 2)}{4} = \frac{2+2}{4} = 1$ M1 A1

(b) $1 + \sinh^2 x = 4\sinh^2 x$ $3\sinh^2 x = 1$ $\sinh x = \pm\frac{1}{\sqrt{3}}$ M1 A1 A1 7

5. (a) Let $u = x + 3$, so integral is $\int\frac{1}{\sqrt{u^2+4}}\,du = \text{arsinh}\left(\frac{u}{2}\right) + c$ M1 A1 A1

$= \text{arsinh}\left(\frac{x+3}{2}\right) + c$ A1

(b) $[\text{arsinh}\{(x+3)/2\}]_{-3}^{3} = \text{arsinh}\,3 - \text{arsinh}\,0 = \ln(3 + \sqrt{10})$ M1 A1 A1 7

6. (a) Let $u = (\ln x)^n$, $dv = x^2\,dx$ $du = n(\ln x)^{n-1}\,dx$ $v = x^3/3$ M1 A1 A1

$I_n = \frac{1}{3}x^3(\ln x)^n - \int\frac{1}{3}x^3 n(\ln x)^{n-1}dx$ $3I_n = x^3(\ln x)^n - n\int x^3(\ln x)^{n-1}dx$ M1 A1

$3I_n = x^3(\ln x)^n - nI_{n-1}$ A1

(b) $I_0 = x^3/3$ $3I_1 = x^3\ln x - I_0$ $3I_2 = x^3(\ln x)^2 - 2I_1$ B1 B1 B1

Thus $[I_2]_1^{e^2} = \frac{1}{3}\left[x^3(\ln x)^2 - \frac{2}{3}\left(x^3\ln x - \frac{x^3}{3}\right)\right]_1^{e^2}$ M1 A1 A1

$= \frac{1}{3}\left[4e^6 - \frac{2}{3}\left(2e^6 - \frac{e^6}{3}\right) - \frac{2}{9}\right]_1^{e^2} = \frac{1}{27}(26e^6 - 2)$ A1 A1 14

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7. (a) $\frac{dy}{dx} = \frac{2a}{2ap} = \frac{1}{p}$ Normal at P is $y - 2ap = -p(x - ap^2)$	M1 A1 M1	
$y - 2ap = ap^3 - px$	A1	
Similarly, normal at Q is $y - 2aq = aq^3 - qx$	A1	
(b) At N, $ap^3 - px + 2ap = aq^3 - qx + 2aq$	M1	
$(p-q)x = ap^3 - aq^3 + 2ap - 2aq = a(p-q)(p^2 + q^2 + pq) + 2a(p-q)$	M1 A1	
Hence $x = a(p^2 + q^2 + pq + 2)$	A1	
$y = ap^3 - px + 2ap = -ap(q^2 + pq + 2) + 2ap = -apq(p+q)$	M1 A1	
(c) When $p = 1$, putting $q = 1$ gives the limit, as Q tends to P, of the point of intersection of the normals (because we have divided by $p - q$, which is tending to zero)	M1	
Thus the point of intersection is tending to $(a(1+1+1+2), -a(1+1))$,	A1	
i.e. $(5a, -2a)$	A1	14
8. (a) When $y = 0$, $4\cosh t = 5$ $\quad 2e^t + 2e^{-t} - 5 = 0$	B1 M1	
$2e^{2t} - 5e^t + 2 = 0$ $\quad (2e^t - 1)(e^t - 2)$ $\quad t = -\ln 2$ or $\ln 2$	A1 A1	
Then $x = 1 + \sinh t = 1 \pm \frac{3}{4}$ $\quad$ Points are $\left(\frac{1}{4}, 0\right)$ and $\left(\frac{7}{4}, 0\right)$	M1 A1 A1	
(b) Curve sketched, symmetric about $x = 1$	B2	
(c) $\frac{dy}{dx} = \frac{-4\sinh t}{\cosh t} = -4\tanh t$ $\quad$ Where $t = \ln 2$, $\frac{dy}{dx} = \frac{-3}{5/4} = -\frac{12}{5}$	M1 A1 A1	
$\frac{d^2y}{dx^2} = -4\,\text{sech}^3\, t = -\frac{256}{125}$ $\quad \rho = \left(\frac{169}{25}\right)^{3/2} . \frac{-125}{256} = -\frac{2197}{256}$	M1 A1 A1	15